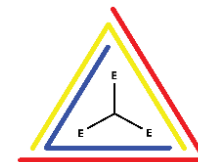




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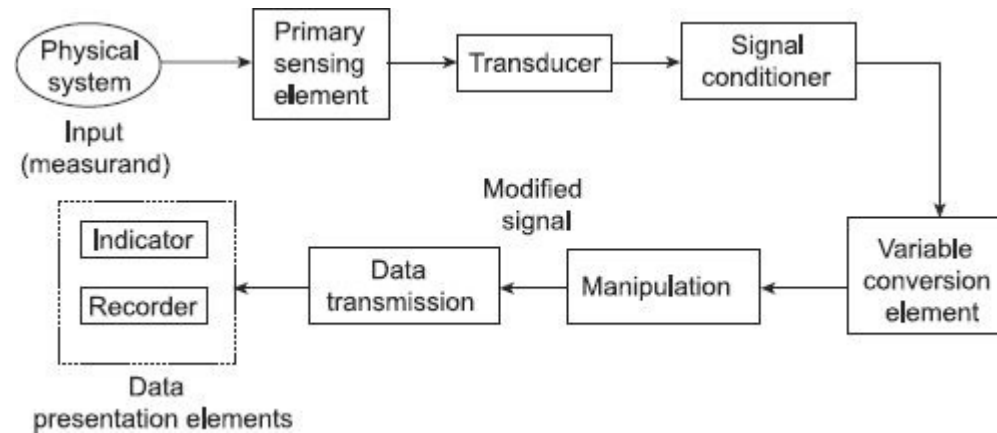
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ELECTRICAL AND ELECTRONICS MEASUREMENT_18EE36 MODULE-1

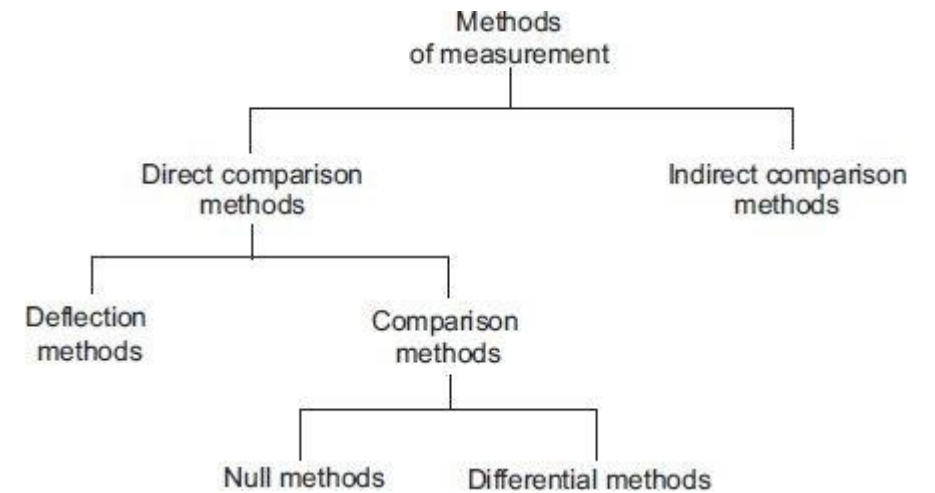
MEASUREMENT OF RESISTANCE, INDUCTANCE AND CAPACITANCE

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MEASUREMENT SYSTEM AND ITS ELEMENTS

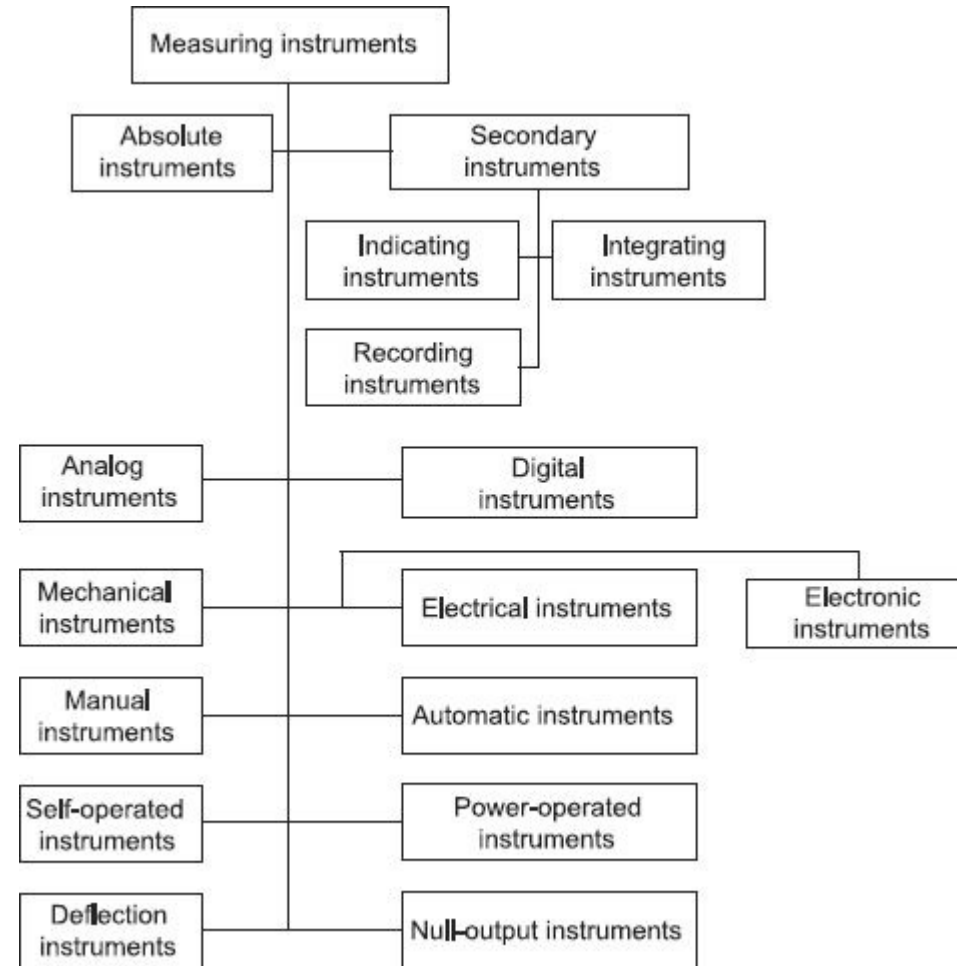


METHODS OF MEASUREMENT



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CLASSIFICATION OF INSTRUMENTS





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MEASUREMENT OF RESISTANCE

LOW, MEDIUM AND HIGH

Classification of Resistances

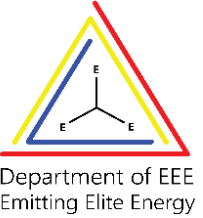
Low Resistances upto and including **one ohm**

Medium Resistances **one ohm upwards** to about **0.1 mega ohm**

High Resistances above **0.1 Mega-ohm**



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Measurement of Low Resistance and Medium Resistance

The methods used for measurement

Ammeter voltmeter method

Kelvin's double bridge method

Potentiometer method

Wheatstone's bridge

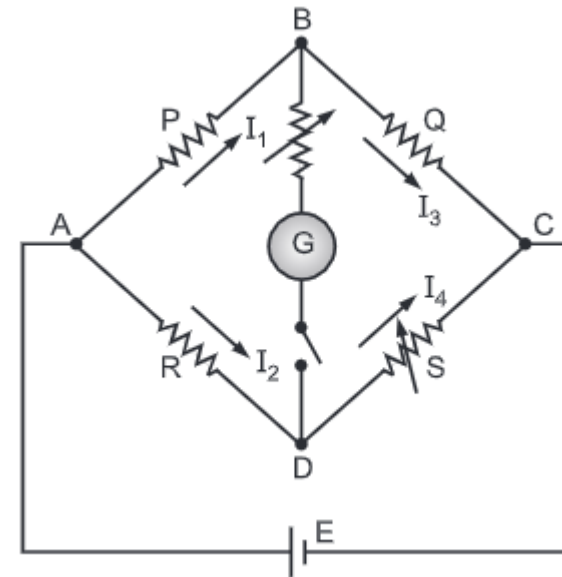
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Wheatstone's Bridge Method

measuring an unknown resistance

medium resistance in industry

very accurate and reliable method of measuring



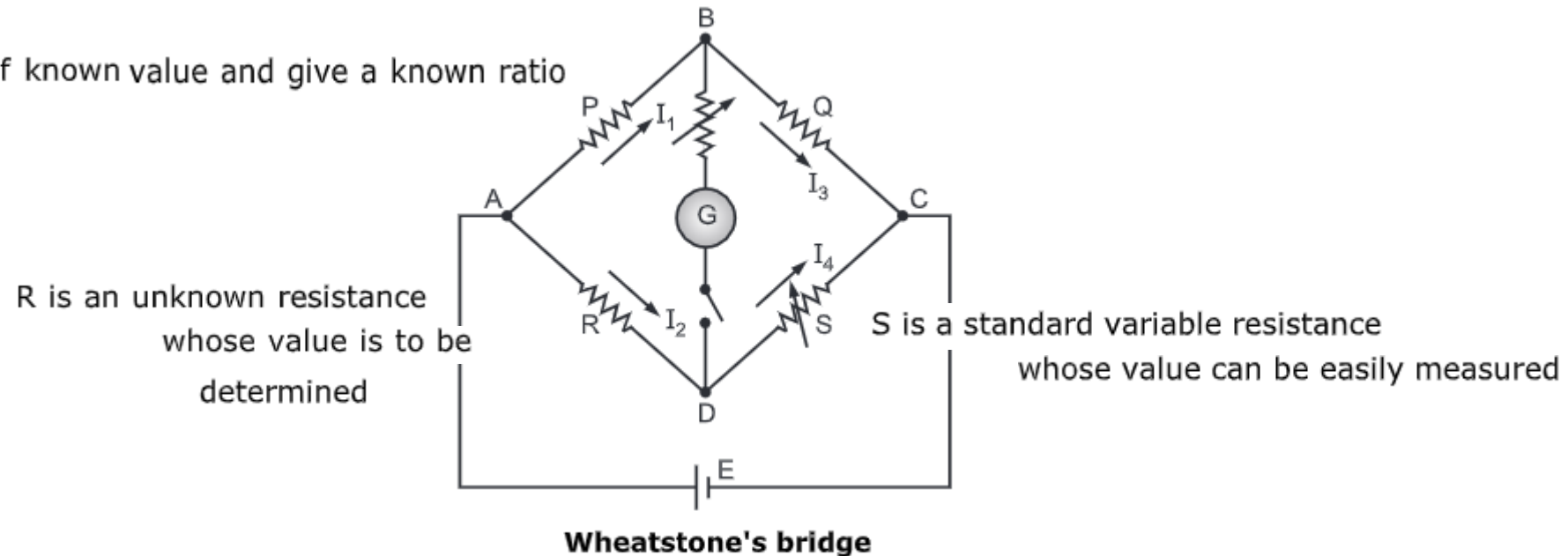
Wheatstone's bridge

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CONSTRUCTION

consists of four resistance arms P, Q, R and S

P and Q are of known value and give a known ratio



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WORKING PRINCIPAL

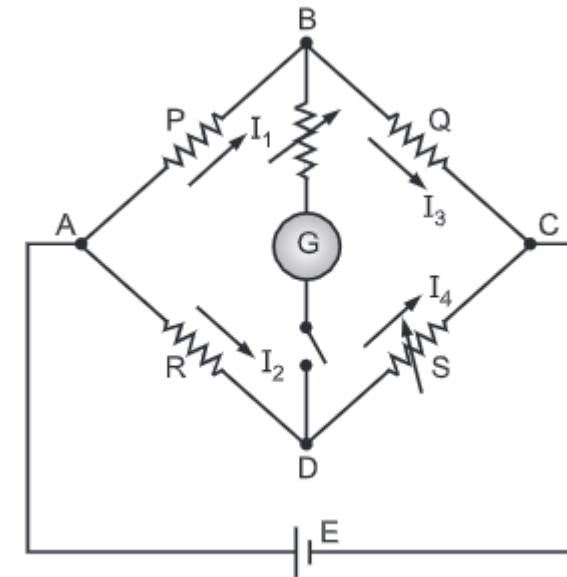
A source of e.m.f. is connected between point AC

galvanometer G is connected between BD

which is used as a null detector

The current flowing through the galvanometer will depend upon

the potential difference between B and D when there is no current through the galvanometer, which will be obtained by adjusting the value of standard resistance



Wheatstone's bridge

Under this condition, voltage from point B to A will be equal to the voltage from point D to point A

or

when the voltage from point D to C is equal to the voltage from point B to C.

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For the balanced condition of bridge, we can write the equations if the assumed current in different branches of Wheatstone's bridge are as shown in Fig

$$I_1 P = I_2 R \quad \dots (1)$$

$$I_1 = I_3$$

$$I_1 = \frac{E}{P + Q} = I_3 \quad \dots (2)$$

$$I_2 = I_4 = \frac{E}{R + S} \quad \dots (3)$$

where, E is e.m.f. of the battery

Combining (1), (2) and (3) and simplifying we get

$$\frac{P}{P + Q} = \frac{R}{R + S}$$

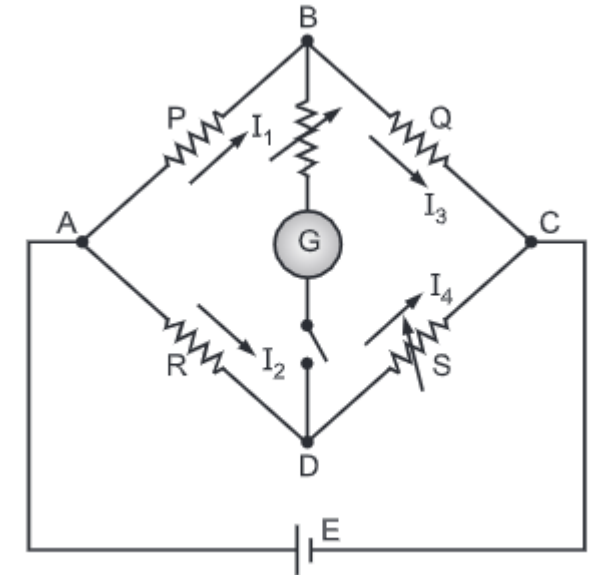
or

$$QR = PS$$

or

$$R = \frac{P}{Q} \times S$$

The unknown can be determined



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Errors in a Wheatstone Bridge

A Wheatstone bridge is a fairly convenient and accurate method for measuring resistance. However, it is not free from errors as listed below:

1. Discrepancies between the true and marked values of resistances of the three known arms can introduce errors in measurement.
2. Inaccuracy of the balance point due to insufficient sensitivity of the galvanometer may result in false null points.
3. Bridge resistances may change due to self-heating (I^2R) resulting in error in measurement calculations.
4. Thermal emfs generated in the bridge circuit or in the galvanometer in the connection points may lead to error in measurement.
5. Errors may creep into measurement due to resistances of leads and contacts. This effect is however, negligible unless the unknown resistance is of very low value.
6. There may also be personal errors in finding the proper null point, taking readings, or during calculations.

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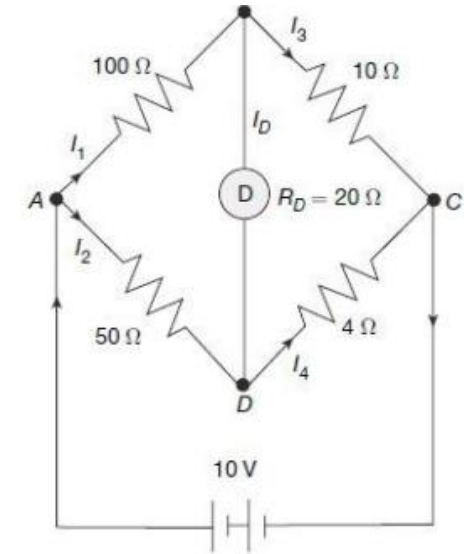
Four arms of a Wheatstone bridge are as follows: $AB = 100 \Omega$, $BC = 10 \Omega$, $CD = 4 \Omega$, $DA = 50 \Omega$. A galvanometer with internal resistance of 20Ω is connected between BD , while a battery of 10-V dc is connected between AC . Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out Thevenin equivalent voltage across nodes BD and also the Thevenin equivalent resistance between terminals BD .

To find out Thevenin's equivalent voltage across BD , the galvanometer is open circuited, and the circuit then looks like the figure given below.

At this condition, voltage drop across the arm BC is given by



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$$V_{BC} = 10 \times \frac{10}{100+10} = 0.91 \text{ V}$$

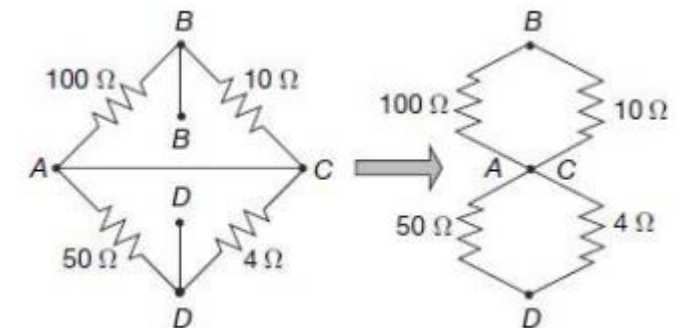
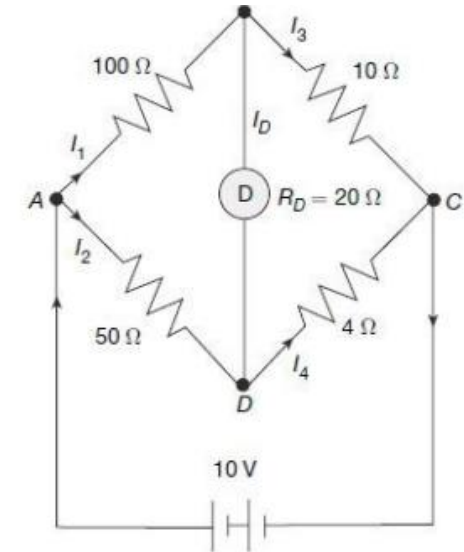
Voltage drop across the arm DC is given by:

$$V_{DC} = 10 \times \frac{4}{50+4} = 0.74 \text{ V}$$

Hence, voltage difference between the nodes B and D , or the Thevenin equivalent voltage between nodes B and D is

$$V_{TH} = V_{BD} = V_B - V_D = V_{BC} - V_{DC} = 0.91 - 0.74 = 0.17 \text{ V}$$

To obtain the Thevenin equivalent resistance between nodes B and D , the 10 V source need to be shorted, and the circuit looks like the figure given below.



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The Thevenin equivalent resistance between the nodes B and D is thus

$$R_{Th} = \frac{100 \times 10}{100 + 10} + \frac{50 \times 4}{50 + 4} = 12.79 \, \Omega$$

Hence, current through galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.17}{20 + 12.79} = 5.18 \, \text{mA}$$

In order to balance the bridge, there should be no current through the galvanometer, or in other words, nodes B and D must be at the same potential.

Balance equation is thus

$$\frac{100}{10} = \frac{R_{DA}}{4} \quad \text{or, } R_{DA} = 40 \, \Omega$$

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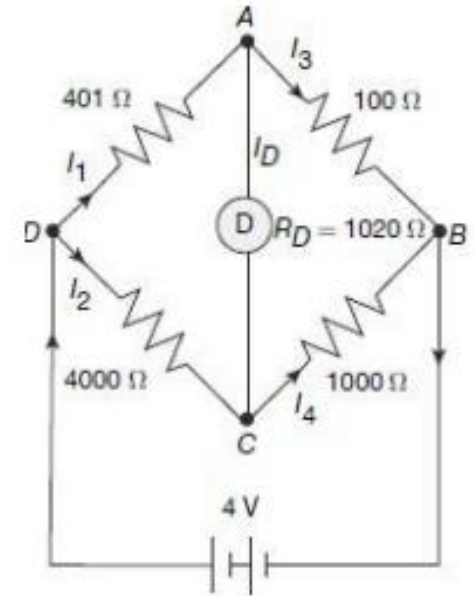
The four arms of a Wheatstone bridge are as follows: $AB = 100 \Omega$, $BC = 1000 \Omega$, $CD = 4000 \Omega$, $DA = 400 \Omega$. A galvanometer with internal resistance of 100Ω and sensitivity of $10 \text{ mm}/\mu\text{A}$ is connected between AC, while a battery of 4 V dc is connected between BD. Calculate the current through the galvanometer and its deflection if the resistance of arm DA is changed from 400Ω to 401Ω .

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out the Thevenin equivalent voltage across nodes AC and also the Thevenin equivalent resistance between terminals AC.

To find out Thevenin equivalent voltage across AC, the galvanometer is open circuited. At this condition, voltage drop across the arm AB is given by

$$V_{AB} = 4 \times \frac{100}{100 + 401} = 0.798 \text{ V}$$



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Hence, voltage difference between the nodes A and C , or the Thevenin equivalent voltage between nodes A and C is

$$\begin{aligned} V_{TH} &= V_{AC} \\ &= V_A - V_C \\ &= V_{AB} - V_{CB} \\ &= 0.798 - 0.8 = -0.002 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Deflection of the galvanometer} &= \text{Sensitivity} \times \text{Current} \\ &= 10 \text{ mm}/\mu\text{A} \times 2.04 \mu\text{A} \\ &= 20.4 \text{ mm} \end{aligned}$$

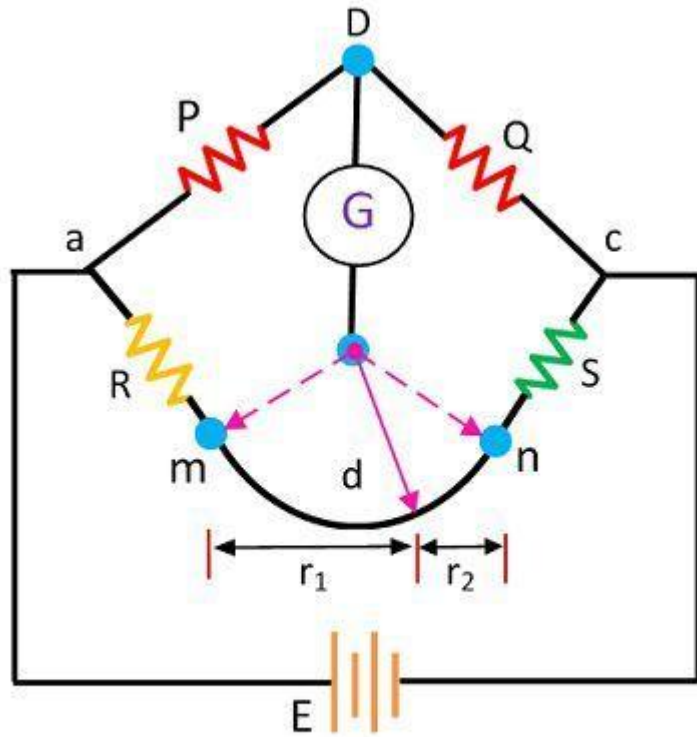
To obtain the Thevenin equivalent resistance between nodes A and C , the 10 V source need to be shorted. Under this condition, the Thevenin equivalent resistance between the nodes A and C is thus

$$R_{Th} = \frac{100 \times 401}{100 + 401} + \frac{1000 \times 4000}{1000 + 4000} = 880.04 \Omega$$

Hence, current through the galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.002}{100 + 880.04} = 2.04 \mu\text{A}$$

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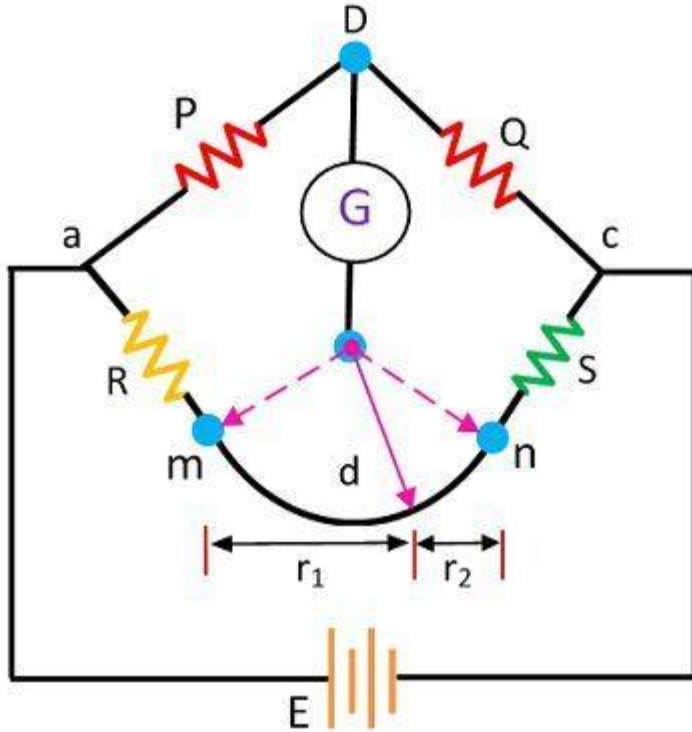


Prinicple of Kelvin's Bridge

The Kelvin bridge or Thompson bridge is used for measuring the unknown resistances having a value less than 1Ω . It is the modified form of the Wheatstone Bridge

Wheatstone bridge use for measuring the resistance from a few ohms to several kilo-ohms. **But error occurs in the result when it is used for measuring the low resistance.** This is the reason because of which the [Wheatstone bridge](#) is modified, and the Kelvin bridge obtains. The Kelvin bridge is suitable for measuring the low [resistance](#).

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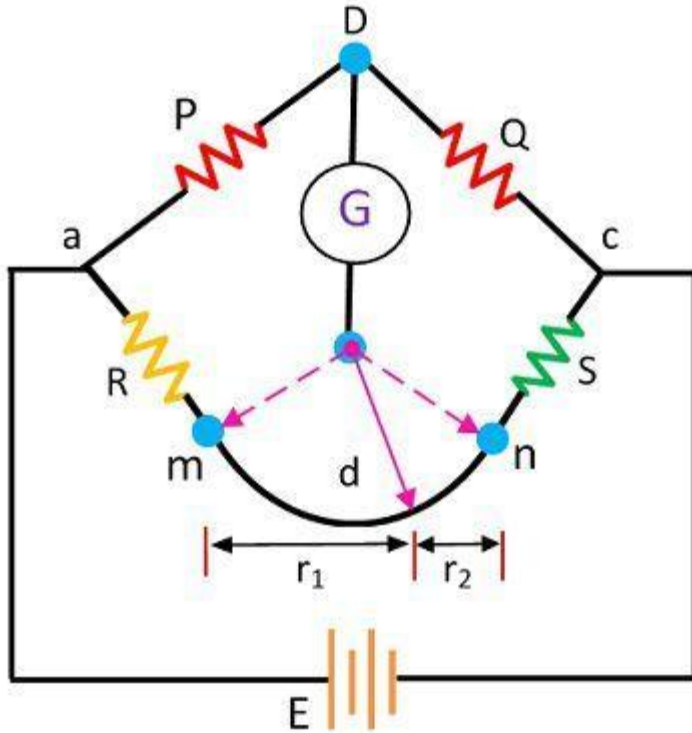


Prinicple of Kelvin's Bridge

In Wheatstone Bridge, while measuring the low-value resistance, the resistance of their lead and contacts increases the resistance of their total measured value. This can easily be understood with the help of the circuit diagram.

The r is the resistance of the contacts that connect the **unknown resistance R** to the **standard resistance S** . The ' m ' and ' n ' show the range between which the galvanometer is connected for obtaining a null point

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Prinicple of Kelvin's Bridge

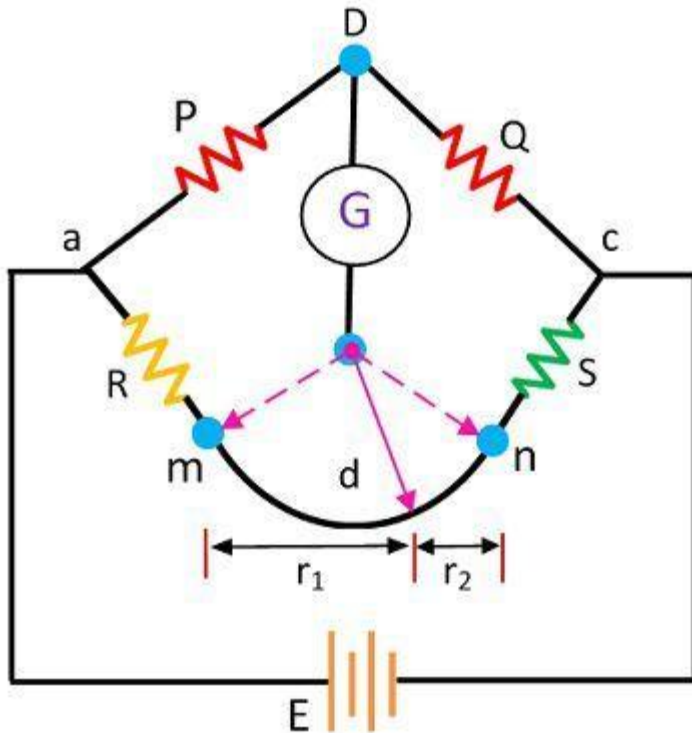
When the galvanometer is connected to point 'm', the lead resistance r is added to the standard resistance S . Thereby the very low indication obtains for unknown resistance R .

And if the galvanometer is connected to point n then the r adds to the R , and hence the high value of unknown resistance is obtained. Thus, at point n and m either very high or very low value of unknown resistance is obtained.

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So, instead of connecting the galvanometer from point, m and n we chose any intermediate point say d, Then $R + r_1 = \frac{P}{Q} \cdot (S + r_2)$

where the resistance of lead r is divided into two equal parts, i.e., r_1 and r_2 $\frac{r_1}{r_2} = \frac{P}{Q} \dots \dots \dots equ(1)$



Prinicple of Kelvin's Bridge

From equation (1), we get

$$\frac{r_1}{r_1 + r_2} = \frac{P}{P + Q}$$

$$r_1 = \frac{P}{P + Q} \cdot r$$

as

$$r_1 + r_2 = r$$

$$r_2 = \frac{Q}{P + Q} \cdot r$$

$$R + \frac{P}{P + Q} \cdot r = \frac{P}{Q} \left(S + \frac{Q}{P + Q} r \right)$$

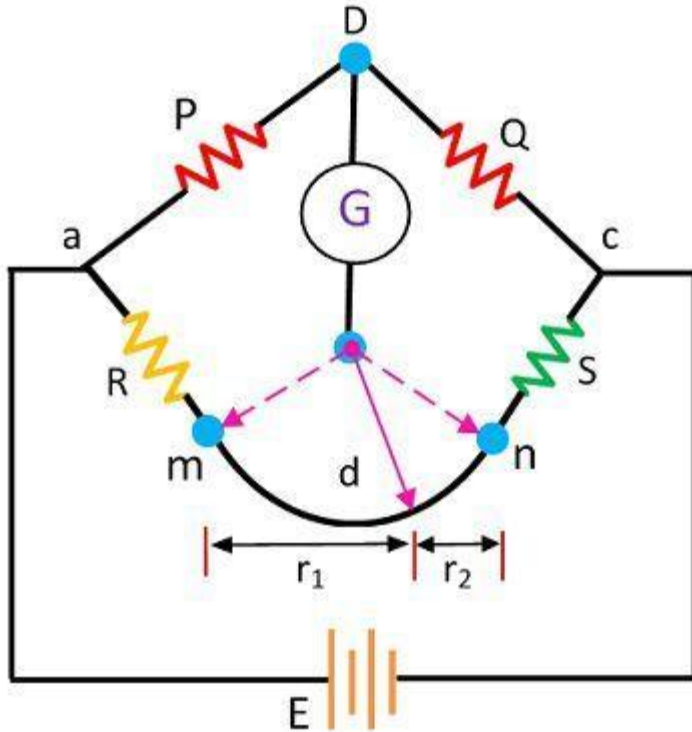
$$R = \frac{P}{Q} \cdot S$$

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$$R = \frac{P}{Q} \cdot S$$

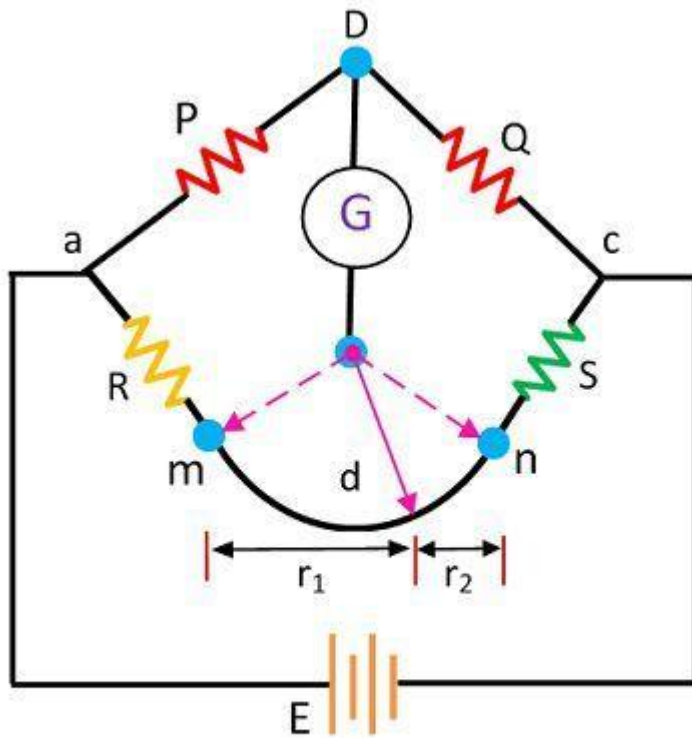
The above equation shows that if the galvanometer connects at point d then the resistance of lead will not affect their results.

The above mention process is practically not possible to implement. For obtaining the desired result, the actual resistance of exact ratio connects between the point m and n and the galvanometer connects at the junction of the resistor.

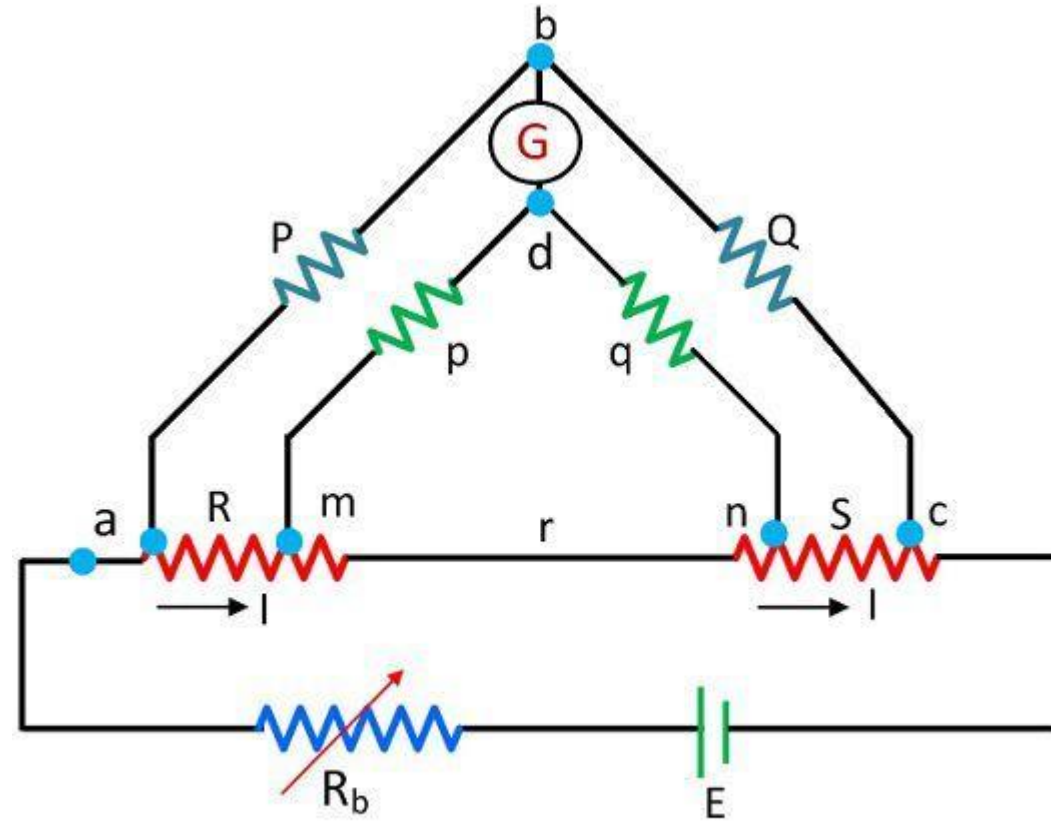


Principle of Kelvin's Bridge

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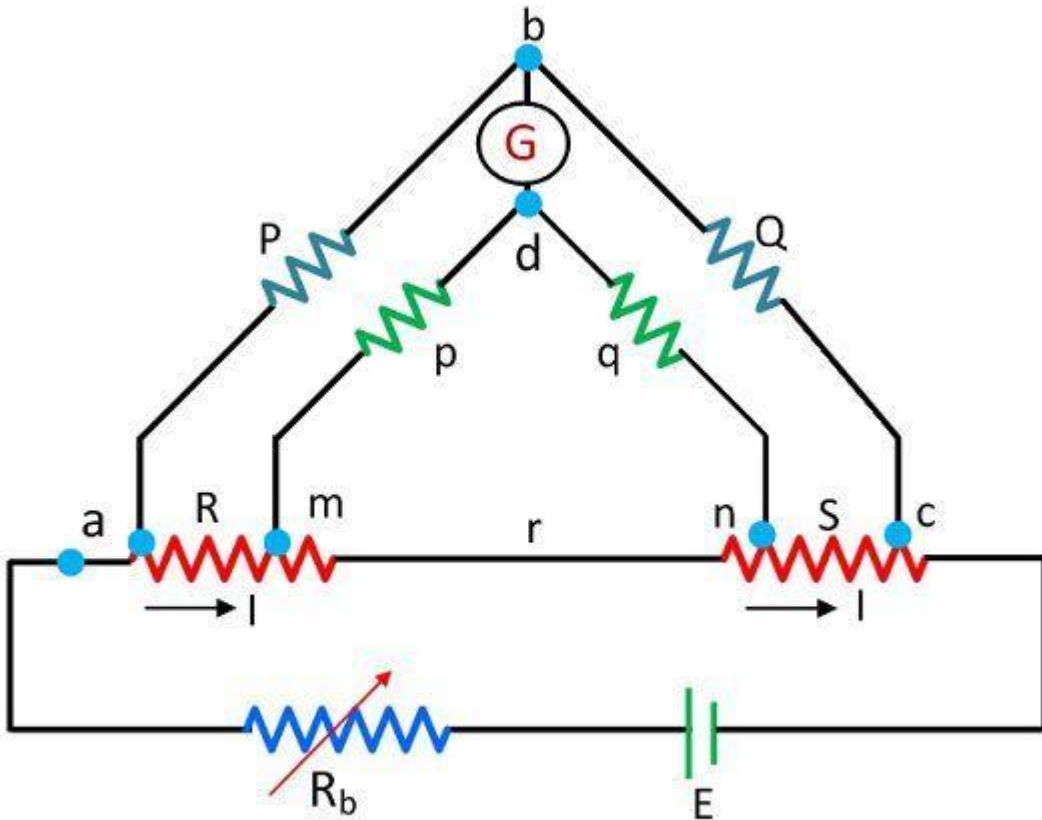


Principle of Kelvin's Bridge



Kelvin Double Bridge

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Kelvin Double Bridge

Circuit Globe

It is used to measure low resistance

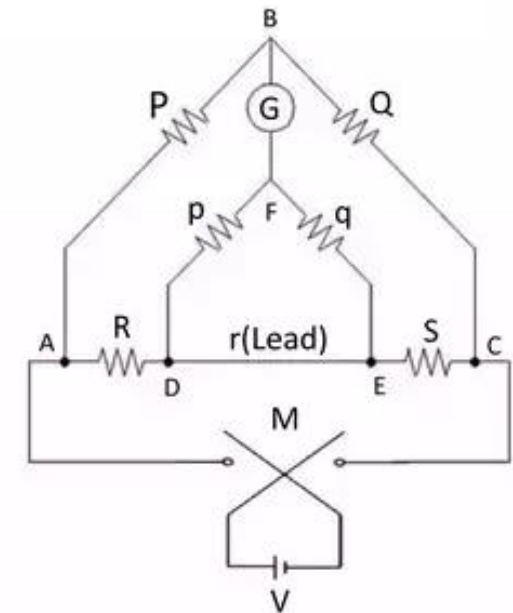
$P, Q \rightarrow$ Outer ratio arms

$R \rightarrow$ Unknown Resistance

$S \rightarrow$ Standard Resistance

$p, q \rightarrow$ Inner ratio arms

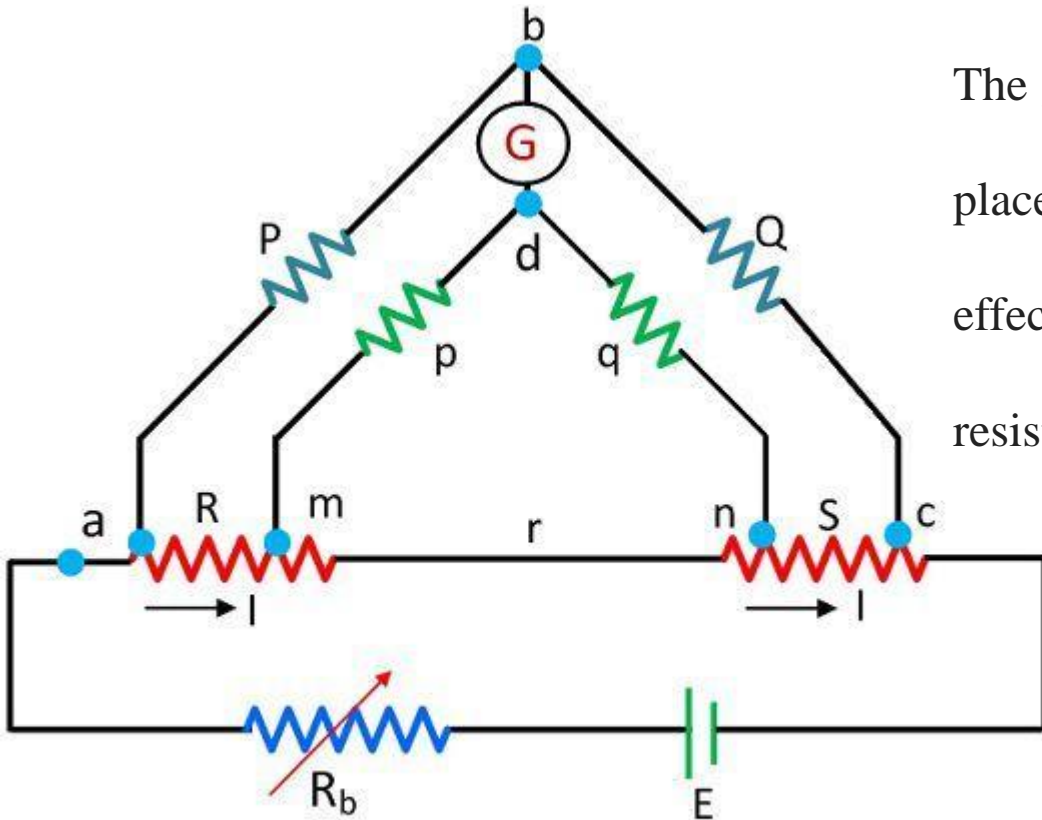
$r \rightarrow$ Lead Resistance



M- Reversible Switch

Kelvin's Double - Bridge Method

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Kelvin Double Bridge

Circuit Globe

The galvanometer is connected between the arms p and q at a point d. The point d places at the centre of the resistance r between the point m and n for removing the effect of the connecting lead resistance which is placed between the unknown resistance R and standard resistance S.

The ratio of p/q is made equal to the P/Q

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At balance, no current flows through the galvanometer

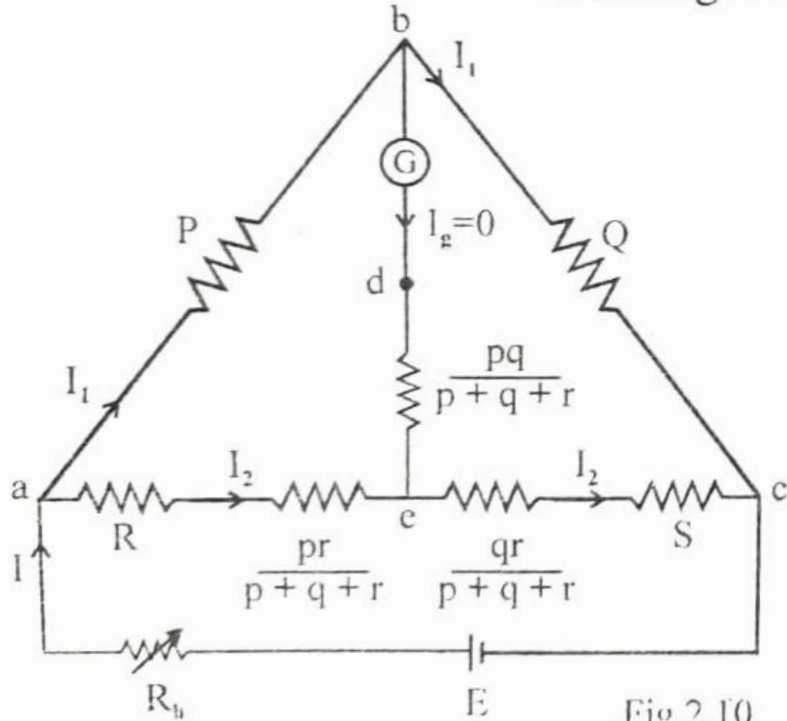


Fig.2.10

$$\therefore V_{ab} = V_{ae}$$

$$\text{i.e. } P I_1 = \left(R + \frac{pr}{p+q+r} \right) I_2$$

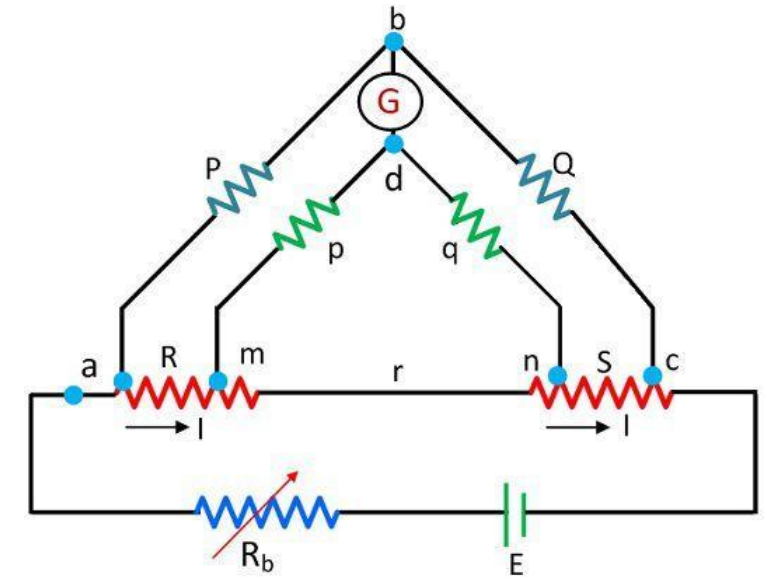
$$\therefore V_{cb} = V_{ce}$$

$$\text{i.e. } Q I_1 = \left(S + \frac{qr}{p+q+r} \right) I_2$$

$$\therefore \frac{P}{Q} = \frac{R + \frac{pr}{p+q+r}}{S + \frac{qr}{p+q+r}} \quad \text{or}$$

$$R = \frac{P}{Q} S + \frac{P}{Q} \frac{qr}{p+q+r} - \frac{pr}{p+q+r}$$

$$= \frac{P}{Q} S + \frac{qr}{p+q+r} \left(\frac{P}{Q} - \frac{p}{q} \right)$$



Kelvin Double Bridge

Circuit Globe

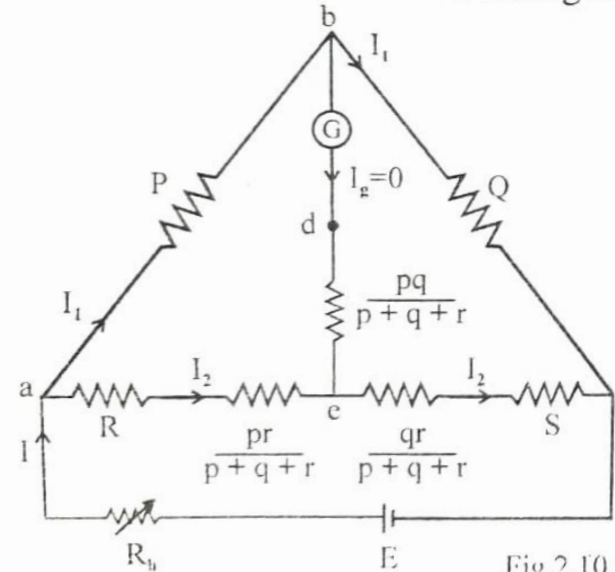
If the ratio of $P/Q = p/q$, Then $R = \frac{P}{Q} \cdot S$

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Limitations of Kelvins Bridge

1. The sensitive galvanometer is used for detecting the balance.
2. The high measurement current is required for obtaining the balance.

At balance, no current flows through the galvanometer



$$\therefore V_{ab} = V_{ac}$$

$$\text{i.e. } P I_1 = \left(R + \frac{pr}{p+q+r} \right) I_2$$

$$\therefore V_{cb} = V_{cc}$$

$$\text{i.e. } Q I_1 = \left(S + \frac{qr}{p+q+r} \right) I_2$$

$$\therefore \frac{P}{Q} = \frac{R + \frac{pr}{p+q+r}}{S + \frac{qr}{p+q+r}} \quad \text{or}$$

$$R = \frac{P}{Q} S + \frac{P}{Q} \frac{qr}{p+q+r} - \frac{pr}{p+q+r}$$

$$= \frac{P}{Q} S + \frac{qr}{p+q+r} \left(\frac{P}{Q} - \frac{p}{q} \right)$$

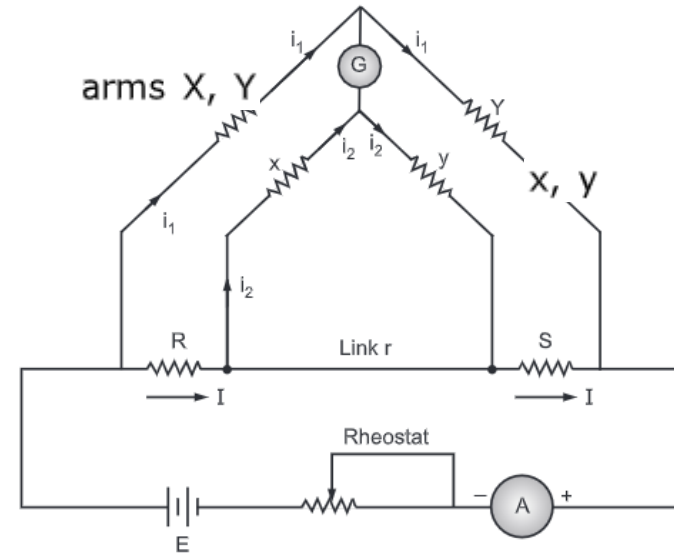
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Kelvin's Double - Bridge Method

A galvanometer is connected between the dividing points
XY and xy

selected that $\frac{X}{Y} = \frac{x}{y}$ under this condition the galvanometer will give zero deflection.

resistance 'R' is an unknown resistance
whose value is to be calculated



consists of a standard
resistance 'S' the value of which can be changed

Kelvin's Double Bridge

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At zero deflection,

$$\frac{R}{S} = \frac{X}{Y} = \frac{x}{y}$$

or

$$R = \frac{X}{Y} S$$

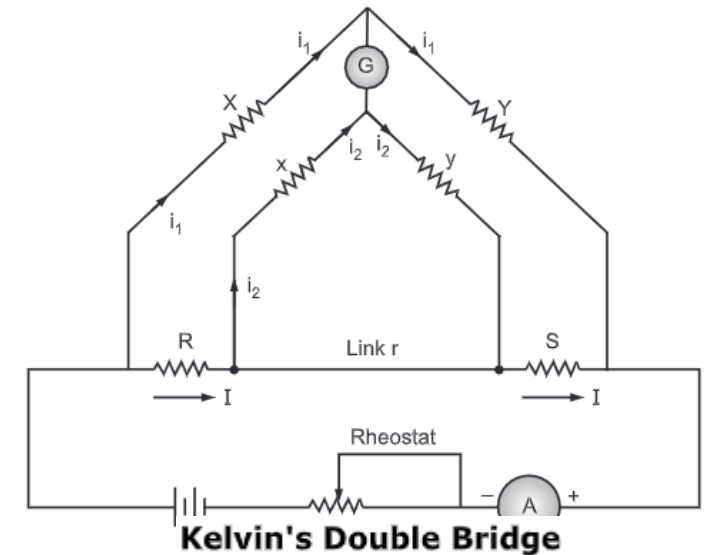
using Kirchhoff's laws

- Let the value of current flowing through branch X = current flowing through branch Y be equal to i_1 amperes; at zero deflection of galvanometer.
- Current through R = Current through S = I. Also from voltage drop across X = voltage drop across R + voltage drop across x.

$$i_1 X = IR + i_2 x \quad \dots(1)$$

and

$$i_1 Y = IS + i_2 y \quad \dots (2)$$



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As x and y are in parallel with resistance r ,

$$i_2 = \frac{r}{(r + x + y)} I \quad \dots(3)$$

Substituting the value of i_2 in equations (1) and (2), we get

$$i_1 X = IR + \frac{r}{(r + x + y)} I \quad \dots(4)$$

$$i_1 Y = IS + \frac{r}{(r + x + y)} \times I \quad \dots(5)$$

Taking ratio of (4) and (5), we get

$$\frac{X}{Y} = \frac{R + \frac{rx}{r + x + y}}{S + \frac{ry}{r + x + y}} \quad \dots (6)$$

Cross multiplying (6),

$$XS + X \frac{ry}{r + x + y} = RY + Y \frac{rx}{r + x + y}$$

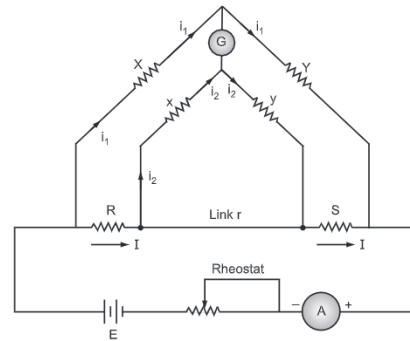
$$RY = XS + \frac{Xry}{(r + x + y)} - \frac{Yrx}{(r + x + y)}$$

$$= XS + \frac{r}{(r + x + y)} (Xy - Yx)$$

unknown resistance,

$$R = \frac{XS}{Y} + \frac{r}{(r + x + y)} \left(\frac{Xy}{Y} - x \right)$$

$$R = \frac{XS}{Y} + \frac{ry}{(r + x + y)} \left(\frac{X}{Y} - \frac{x}{y} \right)$$



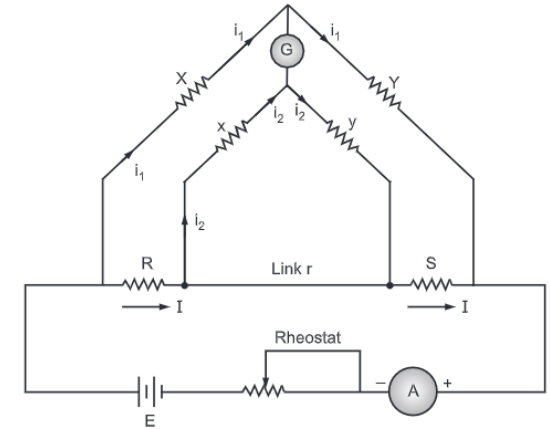
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As r is having a very small value

$\frac{X}{Y} = \frac{x}{y}$ the value $\frac{ry}{(r+x+y)} \left(\frac{X}{Y} - \frac{x}{y} \right)$ will be equal to zero

$$\therefore R = \frac{X}{Y} \times S$$

- The second reading is taken by reversing the direction of flow of current so as to avoid the error that can be introduced by thermoelectric effects of junction. The mean of the two readings will then give the correct value of R .



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A 4-terminal resistor was measured with the help of a Kelvin's double bridge having the following components:

Standard resistor = 98.02 nW, inner ratio arms = 98.022 Ω

and 202 W, outer ratio arms = 98.025 Ω and 201.96 W, resistance of the link connecting the standard resistance and the unknown resistance = 600 nW. Calculate the value of the unknown resistance.

Solution From Eq. value of the unknown resistance is

$$X = \frac{P}{Q} \times S + \frac{qr}{p+q+r} \left(\frac{P}{Q} - \frac{p}{q} \right)$$

$$X = \frac{98.025}{201.96} \times 98.02 \times 10^{-6} + \frac{202 \times 600 \times 10^{-6}}{98.022 + 202 + 600 \times 10^{-6}} \left(\frac{98.025}{201.96} - \frac{98.022}{202} \right)$$

$$X = 47.62 \mu\Omega$$

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Numerical: In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridge uses,

Standard resistance = 100.03Ω

Inner ratio arms = 100.31Ω and 200Ω

Outer ratio arms = 100.24Ω and 200Ω

The resistance of the connecting leads from standard to unknown resistance is $700 \mu \Omega$. Calculate the unknown resistance under this condition.

$$R_3 = 100.03 \mu \Omega, R_2 = 100.24 \Omega, R_1 = 200 \Omega$$

$$b = 100.31 \Omega, a = 200 \Omega, R_y = 700 \mu \Omega$$

Thus unknown resistance is,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{[R_y + a + b]} \left\{ \frac{R_1}{R_2} - \frac{a}{b} \right\}$$

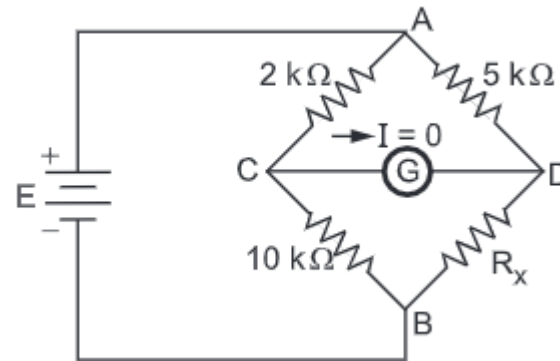
$$= \frac{200 \times 100.03 \times 10^{-6}}{100.24} + \frac{100.31 \times 700 \times 10^{-6}}{[700 \times 10^{-6} + 200 + 100.31]} \left\{ \frac{200}{100.24} - \frac{200}{100.31} \right\}$$

$$= 1.9958 \times 10^{-4} + (2.3381 \times 10^{-4}) (1.3923 \times 10^{-3})$$

$$= 1.999 \times 10^{-4} \Omega = 199.905 \mu \Omega$$

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The Wheatstone bridge is shown in the Fig. Calculate the value of unknown resistance, assuming the bridge to be in balanced condition.



Solution : As per the bridge shown in the Fig.

$$R_1 = 10 \text{ k}\Omega, \quad R_2 = 2 \text{ k}\Omega, \quad R_3 = 5 \text{ k}\Omega \quad \text{and} \quad R_4 = R_x$$

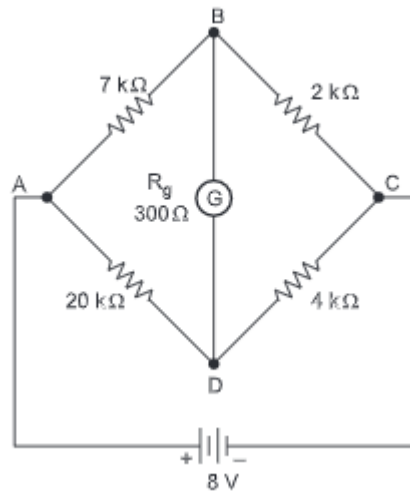
Under balanced condition,

$$R_4 = R_x = \frac{R_1}{R_2} R_3 = \frac{10}{2} \times 5 = 5 \times 5 = 25 \text{ k}\Omega$$

Thus unknown resistance is **25 kΩ** .

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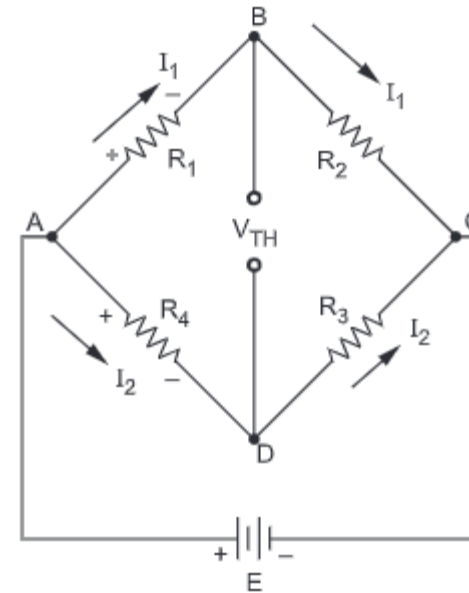
Calculate the current through the galvanometer for the bridge shown in the Fig.



Solution : From the Fig.

$$R_1 = 7 \text{ k}\Omega, \quad R_2 = 2 \text{ k}\Omega, \quad R_3 = 4 \text{ k}\Omega,$$

$$R_4 = 20 \text{ k}\Omega, \quad E = 8 \text{ V}.$$



Use Thevenin's equivalent for I_g .

$$V_{TH} = V_{BD} = V_{AD} - V_{AB} = I_2 R_4 - I_1 R_1$$

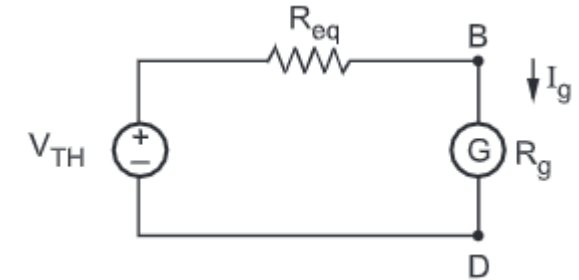
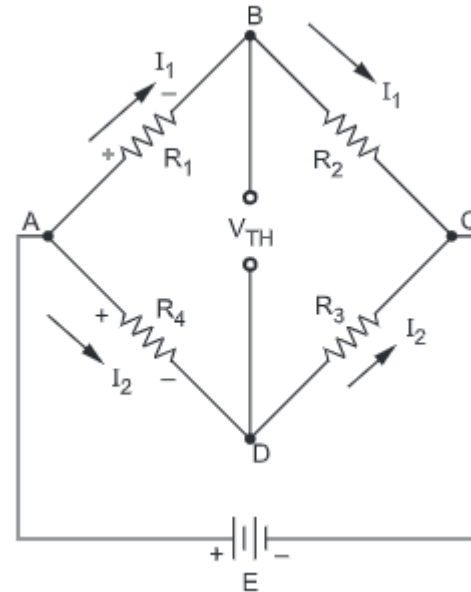
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$$V_{TH} = V_{BD} = V_{AD} - V_{AB} = I_2 R_4 - I_1 R_1$$

$$= \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$= 8 \left\{ \frac{20}{20+4} - \frac{7}{7+2} \right\}$$

$$= 0.444 \text{ V}$$



Now $R_{eq} = [R_1 || R_2] + [R_3 || R_4]$... with E shorted

$$= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 4.888 \text{ k}\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq} + R_g}$$

$$= \frac{0.444}{4.888 \times 10^3 + 300} = 85.62 \text{ }\mu\text{A}$$



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Measurement of Earth Resistance

- The provision of earthing for an electrical system is necessary due to the following reasons:
 - (i) Every metal part in a electrical circuit not carrying any current is required to be earthed as per ISS, so as to make it safe for the workers to work.
 - (ii) At the time of lightening discharge or other faults, those parts of equipment which are normally dead, will attain dangerously high voltages.
 - (iii) In case of three-phase system, the neutral of the system is earthed to stabilize the potential.

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Main Factors Affecting the Resistance of Earthing System

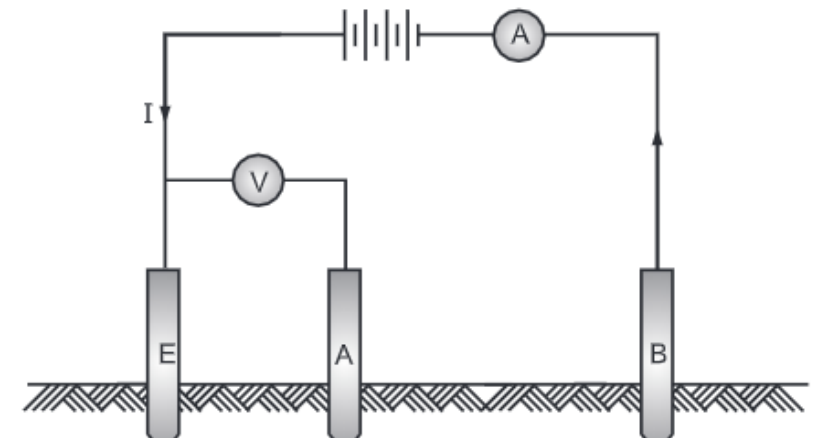
- (a) Shape and material of earth electrode or electrodes used.
- (b) Depth in the soil at which the electrodes are buried.
- (c) Specific resistance of the soil surrounding and in the neighbourhood of electrodes.

Fall of Potential Method of Measuring Earth Resistance

measurement of earth resistance with fall of potential method

- Certain current is made to pass through earth electrode E to an auxiliary electrode B from a battery, which may be in the form of iron spike, inserted in earth at a distance away from E.
- Another electrode A is inserted between E and B. The potential difference V is measured between E and A for the value of current I.

$$\text{The resistance of earth} = \frac{V}{I} = R_E.$$



Measurement of earth resistance

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MEGGER

Resistances of the order of $0.1 \text{ M}\Omega$

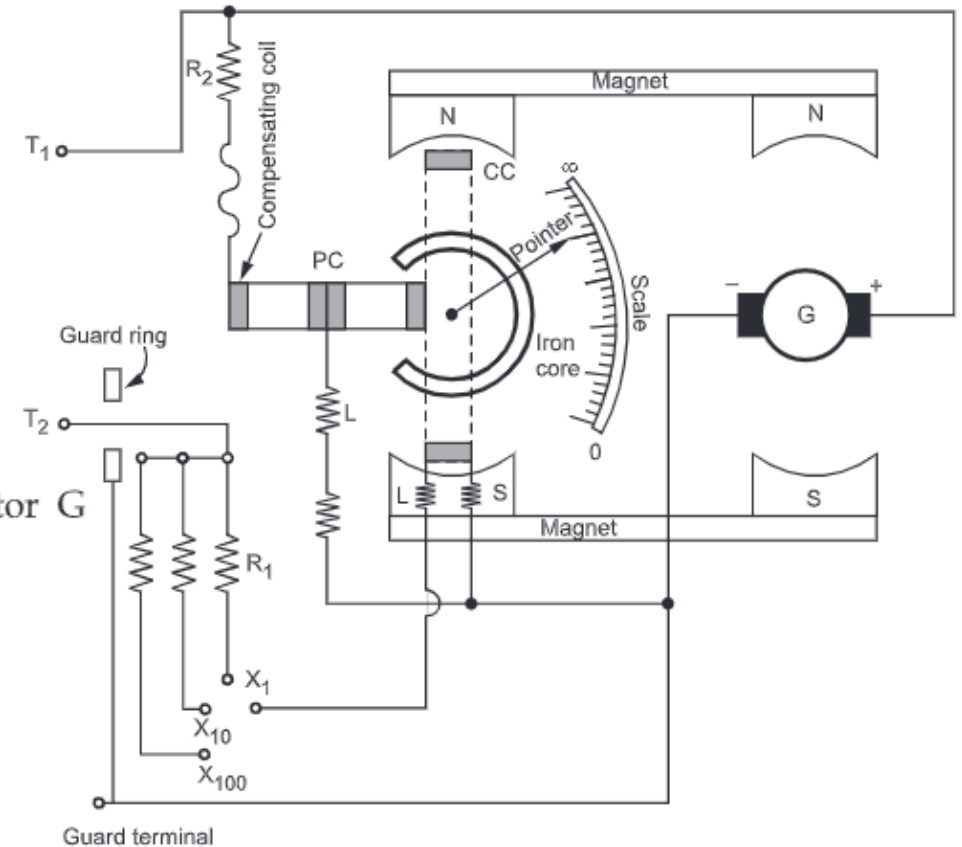
These high resistances are measured by portable, instrument known as **megger**.

It is based on the principle of electromagnetic induction

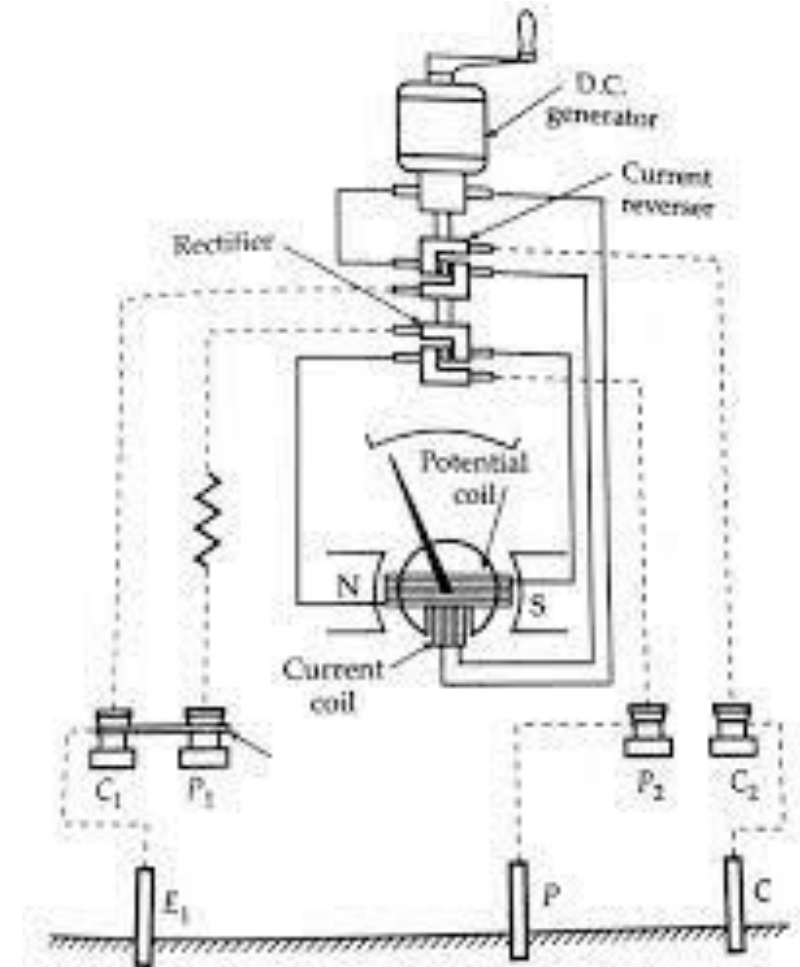
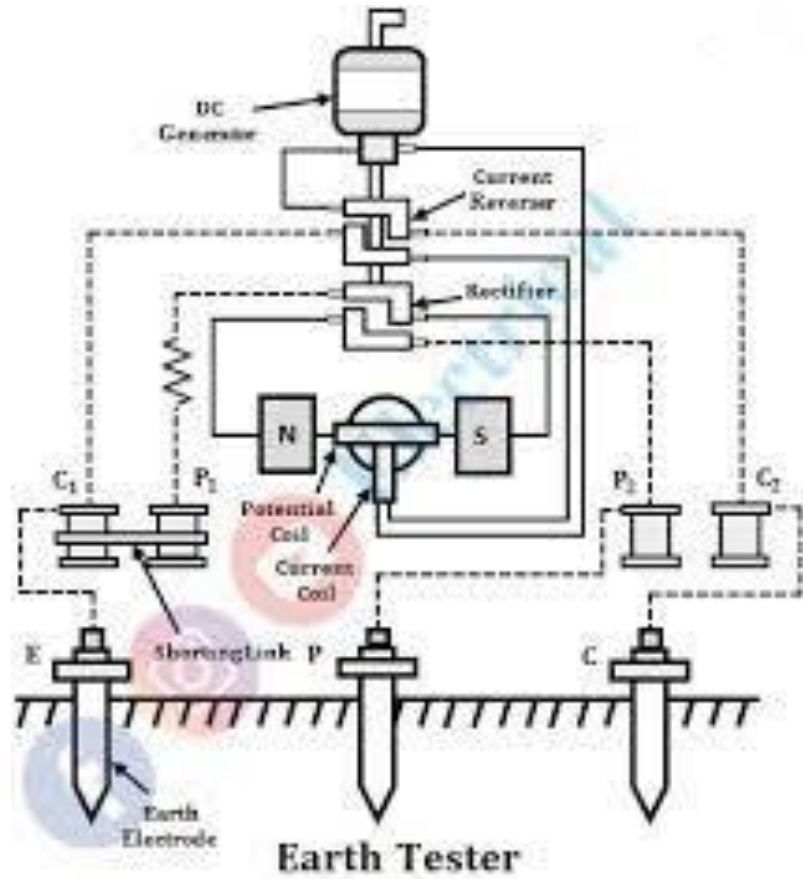
CONSTRUCTION

It consists of a permanent magnet which provides the field for both the generator G and ohmmeter.

ohmmeter consist of three coil
deflection coil,
pressure or control coil
compensating coil



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The current coil is connected in series with resistance R_1 between one generator terminal and the test terminal T_2 .

R_1 protects the current coil in the event of the test terminals getting short circuited

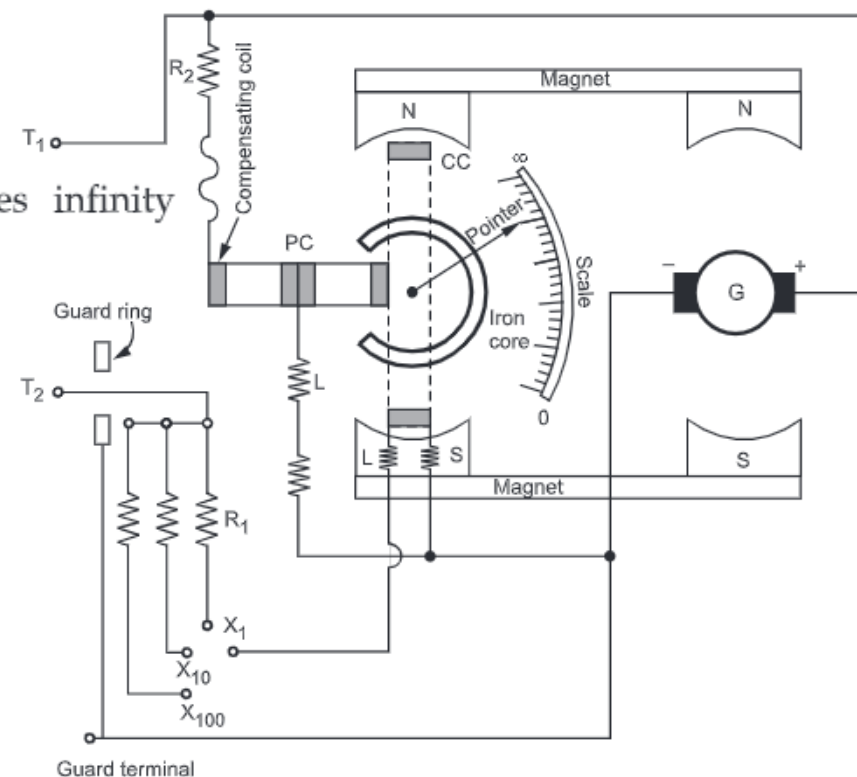
The pressure coil, in series with a compensating coil and protection resistance

R_2 is connected across the generator terminals

compensating coil is included to ensure better scale proportions

The scale is calibrated reversely means the normal position of pointer indicates infinity

while full scale deflection indicates zero resistance.



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WORKING PRINCIPAL

When the current flows from the generator, through the pressure coil, the coil tends to set itself at right angles to the field of the permanent magnet.

When the test terminals are open

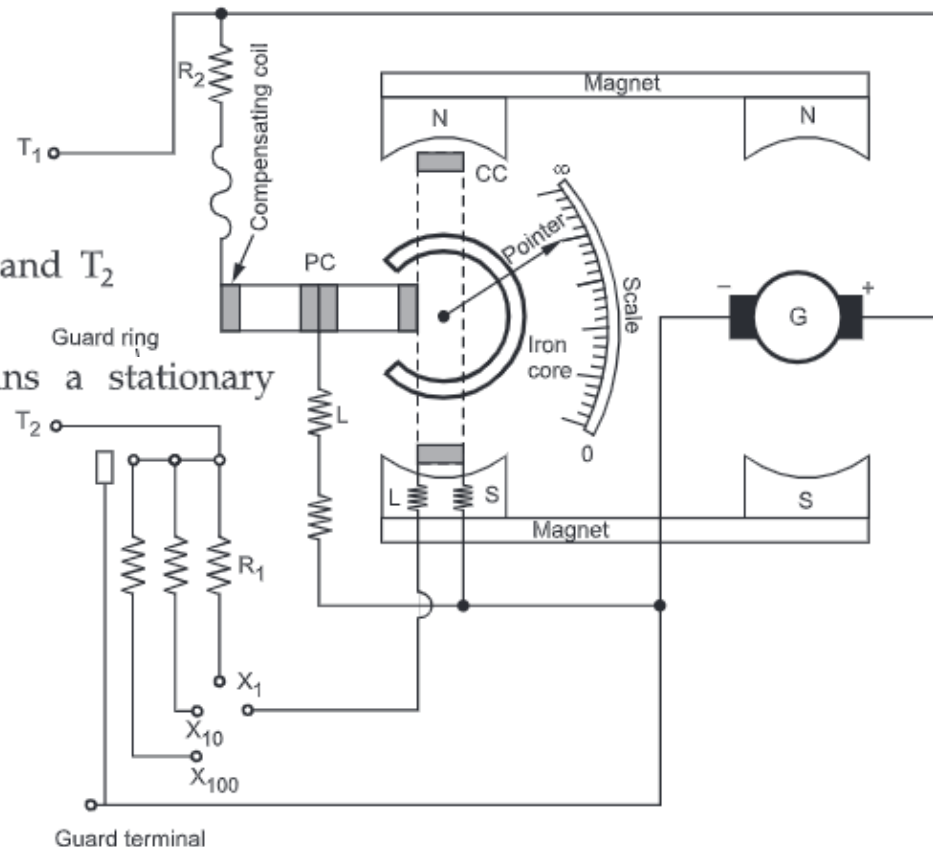
When the test terminals are short circuited

When the high resistance to be tested is connected between terminals T_1 and T_2

opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale.

The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.

guard ring eliminate the error due to leakage current



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The Wheatstone bridge is shown in the Fig. The galvanometer has a current sensitivity of $12 \text{ mm} / \mu\text{A}$.

The internal resistance of galvanometer is 200Ω . Calculate the deflection of the galvanometer caused due to 5Ω unbalance in the arm BD.

Solution : From the given bridge,

$$R_1 = 100 \Omega \quad R_2 = 1000 \Omega$$

$$R_3 = 200 \Omega \quad R_4 = 2000 \Omega$$

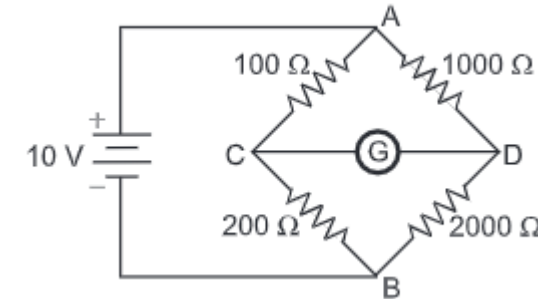
Now $R_1 R_4 = 100 \times 2000 = 200000$

$$R_2 R_3 = 200 \times 1000 = 200000.$$

For $R_4 = 2000 \Omega$, the bridge is balanced. But there is unbalance of 5Ω in the resistance of arm BD i.e. R_4 .

$$R_4 = 2000 + 5 = 2005 \Omega$$

By Thevenin's equivalent, $V_{TH} = E \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right] = 10 \left[\frac{200}{100 + 200} - \frac{2005}{1000 + 2005} \right] = 10 [0.6667 - 0.6672] = -5.213 \text{ mV}$

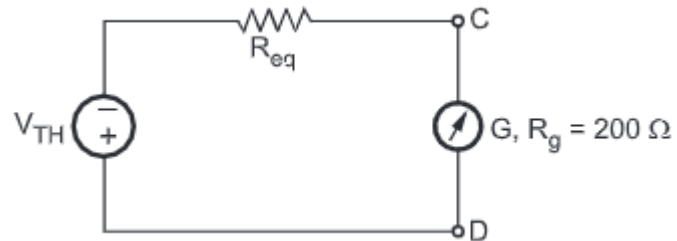


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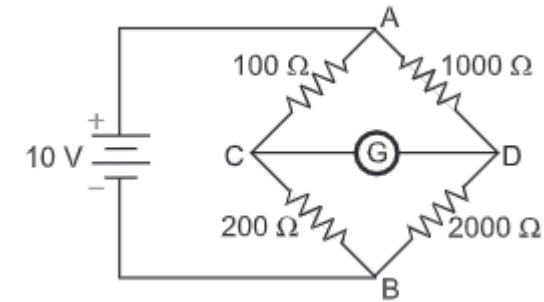
$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{100 \times 200}{(100 + 200)} + \frac{1000 \times 2005}{(1000 + 2005)} = 733.888 \, \Omega$$

Hence Thevenin's equivalent is,



$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{5.213 \times 10^{-3}}{733.888 + 200} = 5.582 \, \mu A$$



Now deflection of galvanometer is proportional to its sensitivity.

$$S = \frac{D}{I}$$

$$D = S \times I$$

$$= 12 \, \text{mm}/\mu A \times 5.582 \, \mu A = 66.98 \, \text{mm}$$

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The four arms of the Wheatstone bridge have the following resistances, $AB = 1000 \Omega$, $BC = 1000 \Omega$, $CD = 120 \Omega$, $DA = 120 \Omega$. The bridge is used for strain measurement and supplied from 5 V ideal battery. The galvanometer has sensitivity of $1 \text{ mm}/\mu\text{A}$ with internal resistance of 200Ω . Determine the deflection of the galvanometer if arm DA increases to 121Ω and arm CD decreases to 119Ω .

Solution : The bridge given is shown in the Fig.

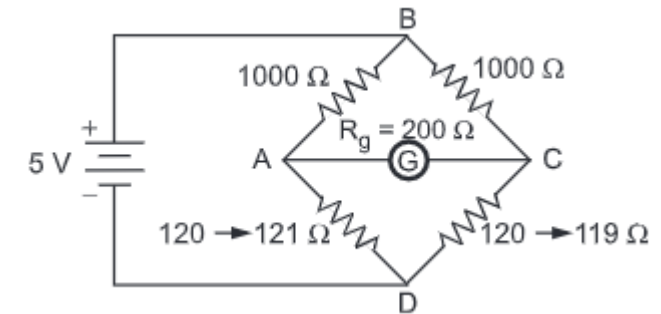
$$\begin{aligned} \text{Now } R_1 &= 1000 \Omega & R_2 &= 1000 \Omega \\ R_3 &= 121 \Omega & R_4 &= 119 \Omega \end{aligned}$$

Let us calculate Thevenin's equivalent due to change in R_3 and R_4 .

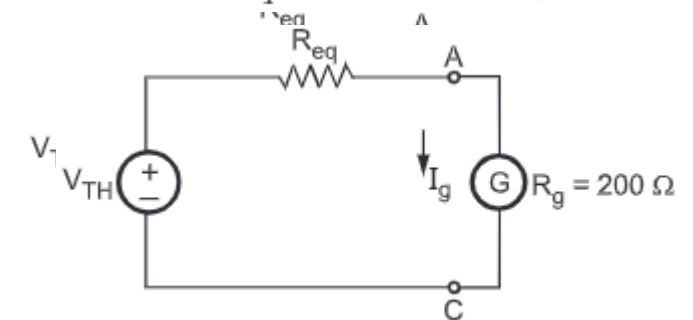
$$V_{TH} = E \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right] = 5 \left[\frac{121}{1000 + 121} - \frac{119}{1000 + 119} \right] = 5 [0.1079 - 0.1063] = 7.975 \text{ mV}$$

$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} = 214.2842 \Omega$$

$$= \frac{121 \times 1000}{121 + 1000} + \frac{119 \times 1000}{119 + 1000} = 107.9393 + 106.3449$$



Thevenin's equivalent circuit is,



$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = 19.24 \mu\text{A} \quad S = \frac{D}{I} = 19.24 \text{ mm}$$

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Using the approximation of slightly unbalanced bridge, calculate the current through the galvanometer having internal resistance of 125Ω , for the bridge shown in the Fig.

Solution : For the bridge shown,

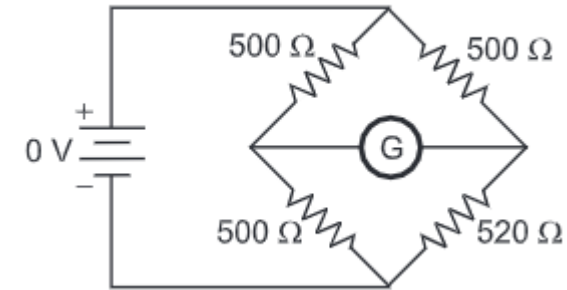
$$R = 500 \Omega \quad \text{and} \quad \Delta r = 20 \Omega$$

Using approximate result,

$$V_{TH} = \frac{E \Delta r}{4 R} = \frac{10 \times 20}{4 \times 500} = 0.1 \text{ V}$$

while $R_{eq} = R = 500 \Omega$

$R_g = 125 \Omega$ give



$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{0.1}{500 + 125} = 160 \mu A$$

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In the Fig. the Kelvin's double bridge is shown. The ratio of R_a to

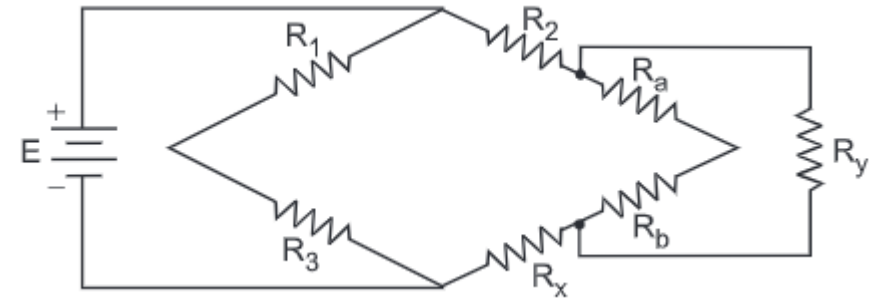
R_b is 1200Ω while R_1 is 10Ω and $R_1 = 0.5 R_2$. Calculate the value of unknown resistance R_x .

From the Fig.
$$\frac{R_x}{R_2} = \frac{R_3}{R_1}$$

For Kelvin's double bridge,

$$\frac{R_3}{R_1} = \text{Ratio of resistances of ratio arms}$$

$$\frac{R_b}{R_a} = \text{Ratio of resistances of second ratio arms}$$



$$\frac{R_x}{R_2} = \frac{R_b}{R_a} = \frac{1}{1200}$$

$$R_2 = \frac{R_1}{0.5} = \frac{10}{0.5} = 20 \Omega$$

$$\frac{R_3}{R_1} = \frac{R_b}{R_a}$$

Now

$$R_1 = 10 \Omega$$

$$R_1 = 0.5 R_2$$

$$\frac{R_x}{20} = \frac{1}{1200} = 0.0167 \Omega$$

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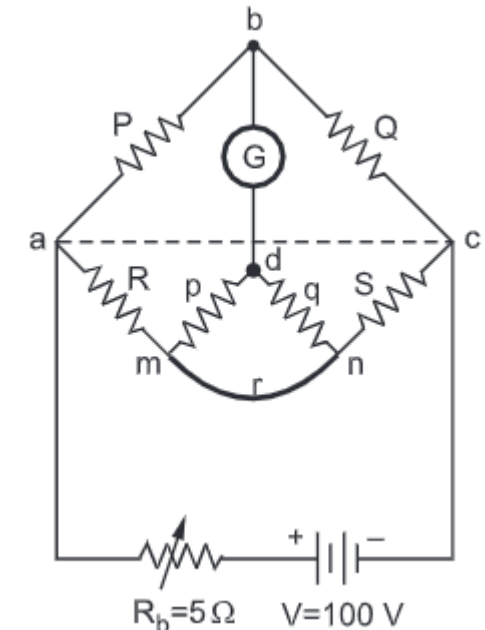
A Kelvin double bridge has each of the ratio arms $P = Q = p = q = 1000 \Omega$. The e.m.f. of the battery is 100 V and resistance of 5Ω is included in the circuit. The galvanometer has a resistance of 500Ω and the resistance of the link connecting the unknown resistance to the standard resistance may be neglected. The bridge is balanced when standard resistance $S = 0.001 \Omega$.

- Determine the value of unknown resistance.
- Determine the current (approximate value) through the unknown resistance and at balance.
- Determine the deflection of the galvanometer when the unknown resistance R is changed by 0.1 % from its value at balance.

The galvanometer has a sensitivity of $200 \text{ mm} / \mu\text{A}$.

Solution : A Kelvin double bridge is as shown in the Fig.

a) At balance,
$$R = \frac{P}{Q} \cdot S = \frac{1000}{1000} (0.001) = 0.001 \Omega$$



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b) Current under balance condition,

$$I = \frac{V}{R_b + R + S} = \frac{100}{5 + 0.001 + 0.001} = 19.99 \text{ A}$$

c) The value of R is changed by 0.1 %.

$$\therefore \text{New value of } R = 0.001 \times 0.1 = 0.0001 \Omega$$

$$V_{ac} = \left[\frac{R + r + S}{R_b + R + r + S} \right] V$$

Neglecting r,

$$V_{ac} = \frac{R + S}{R_b + R + S} V = \frac{0.0001 + 0.001}{5 + 0.0001 + 0.001} \times 100 = 29.995 \text{ mV}$$

$$V_{ab} = \frac{P}{P + Q} V_{ac} = \frac{1000}{1000 + 1000} (29.995 \times 10^{-3}) = 14.9978 \text{ mV}$$

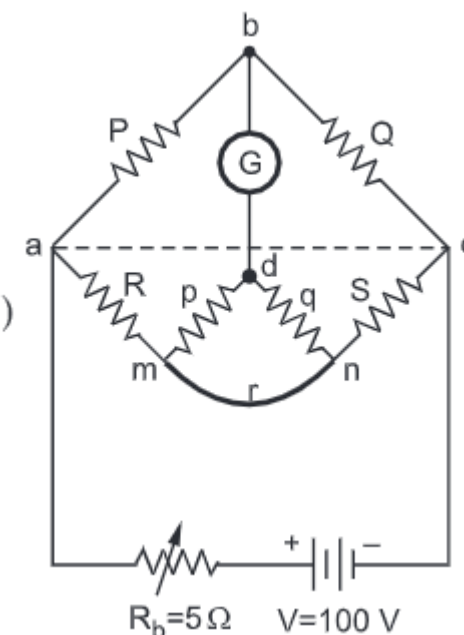
$$V_{amd} = \left[\frac{R + \frac{P_r}{p + q + r}}{R + S + \frac{(p + q)r}{p + q + r}} \right] V_{ab}$$

Neglecting r,

$$\begin{aligned} V_{amd} &= \frac{R}{R + S} V_{ab} \\ &= \frac{0.0001}{0.0001 + 0.001} (14.9978 \times 10^{-3}) \\ &= 1.3634 \text{ mV} \end{aligned}$$

Hence output voltage is given as,

$$\begin{aligned} V_{out} &= V_{ab} - V_{amd} \\ &= 14.9978 \text{ mV} - 1.3634 \text{ mV} \\ &= 0.01362 \text{ V} \end{aligned}$$



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Types of Bridges

The two types of d.c. bridges are,

- 1) D.C. bridges used to measure the **resistances**
- 2) A.C. bridges used to measure the **impedances** consisting capacitances and inductances.

D.C. bridges

1. Wheatstone bridge
2. Kelvin bridge

A.C. bridges

- | | |
|-----------------------------------|---------------------|
| 1. Capacitance comparison bridge. | 4. Hay's bridge. |
| 2. Inductance comparison bridge. | 5. Anderson bridge. |
| 3. Maxwell's bridge. | 6. Schering bridge. |
| | 7. Wien bridge. |

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Sources and Detectors

For bridge measurements

at very low frequencies,

the power line itself may act as a source of supply to the bridge circuit.

at higher frequencies

electronic oscillators are used as a source of supply to the bridge circuit.

- i) The output waveform is very close to sine wave.
- ii) The output frequency is very stable.
- iii) The output frequency is easily determinable with accuracy and also it is easily adjustable.
- iv) The output power is sufficient to drive the bridge circuits.

40 Hz to 125 kHz with power output of 7 W.



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For the a.c. bridges commonly used detectors are as follows.

i) Headphones :

250 Hz upto 3 to 4 kHz.

ii) Vibration galvanometers :

5 Hz to 1000 Hz.

These detectors can be effectively used below 200 Hz with greater sensitivity than the headphones.

iii) Tuneable amplifier detectors :

The frequency range for these detectors is 10 Hz to 100 kHz.

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Maxwell's Inductance Bridge

Using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in Fig.

Two branches consist of non-inductive resistances R_1 and R_2 .

One of the arms consists variable inductance with series resistance r .

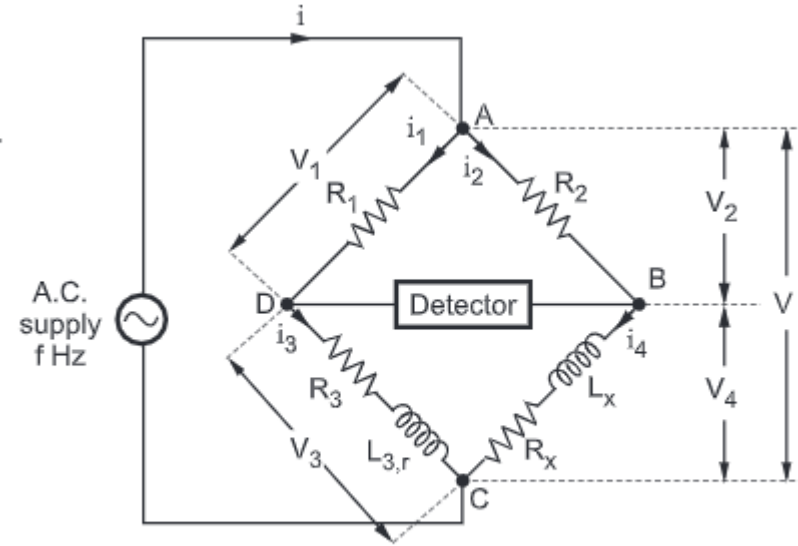
The remaining arm consists unknown inductance L_x .

At balance, we get condition as

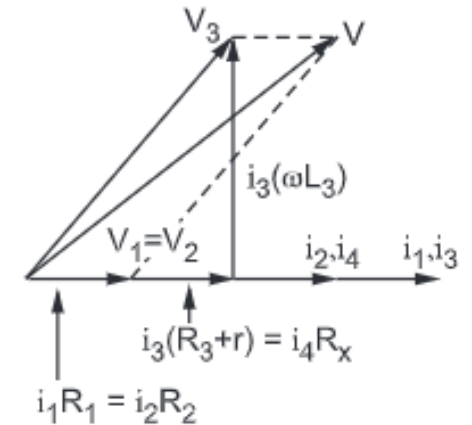
$$\frac{R_1}{[(R_3 + r) + j\omega L_3]} = \frac{R_2}{R_x + j\omega L_x} \quad \dots (1)$$

$$\therefore R_1 [R_x + j\omega L_x] = R_2 [(R_3 + r) + j\omega L_3]$$

$$\therefore R_1 R_x + j\omega R_1 L_x = R_2 (R_3 + r) + j\omega R_2 L_3$$



(a) Circuit diagram



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Equating imaginary terms, we can write

$$R_1 L_x = R_2 L_3$$

$$L_x = \frac{R_3}{R_1} L_3$$

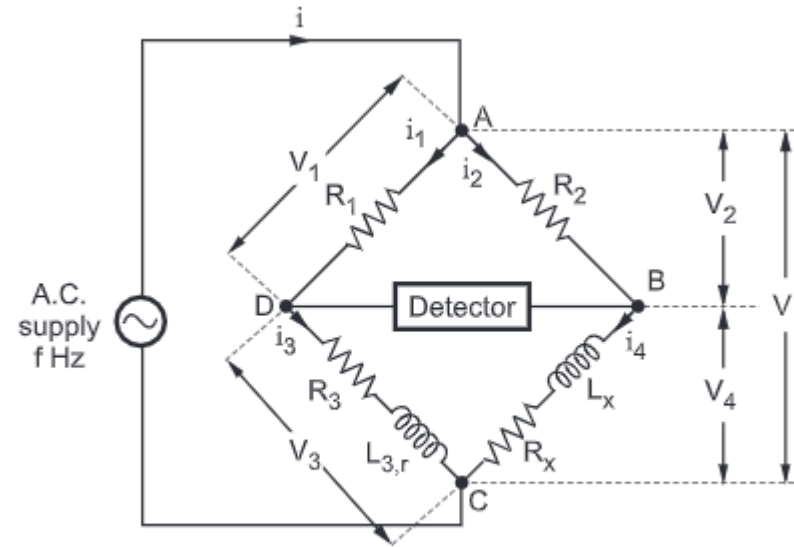
... (2)

Equating real terms, we can write,

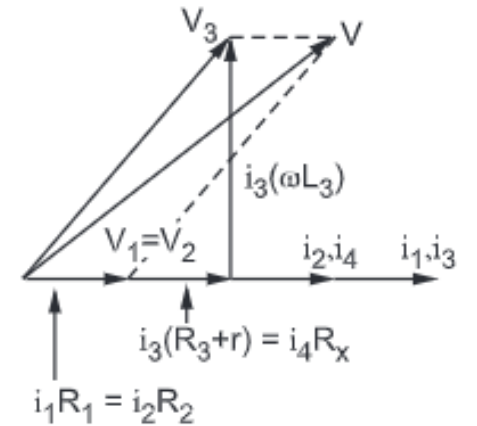
$$R_1 R_x = R_2 (R_3 + r)$$

$$R_x = \frac{R_2}{R_1} (R_3 + r)$$

... (3)



(a) Circuit diagram



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Maxwell's Inductance Capacitance Bridge

Using this bridge, we can measure inductance by comparing with a variable standard capacitor.

The bridge circuit diagram is as shown in the Fig.

One of the ratio arms consists of resistance and capacitance in parallel.

The general bridge balance equation is,

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

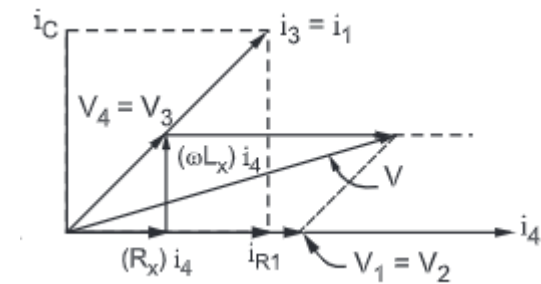
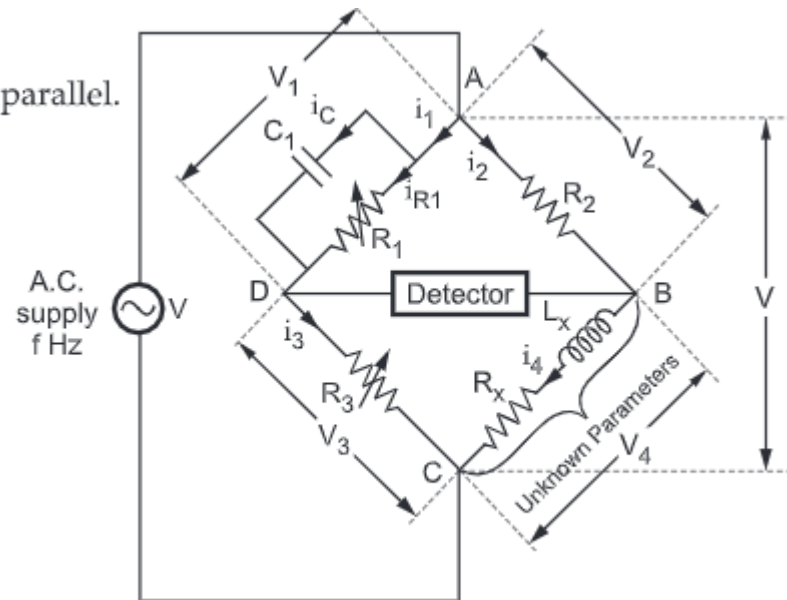
$$\overline{Z_x} = \frac{\overline{Z_2 Z_3}}{\overline{Z_1}} = \overline{Z_2 Z_3 Y_1} \quad \dots (1)$$

where $\overline{Y_1} = \frac{1}{\overline{Z_1}}$ i.e. R_1 in parallel with C_1

$$\overline{Z_2} = R_2$$

$$\overline{Z_3} = R_3$$

$$\overline{Z_x} = R_x + j \omega L_x, \text{ as } L_x \text{ in series with } R_x$$



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Now $\overline{Y_1} = \frac{1}{R_1} + j \omega C_1$

$$\overline{Z_1} = R_1 \parallel j \left(\frac{1}{\omega C_1} \right) \quad \text{as } \frac{1}{j} = -j$$

Substituting all the values in equation (1)

we get,

$$R_x + j \omega L_x = R_2 R_3 \left[\frac{1}{R_1} + j \omega C_1 \right]$$

$$R_x + j \omega L_x = \frac{R_2 R_3}{R_1} + j R_2 R_3 \omega C_1$$

Equating real parts,

$$R_x = \frac{R_2 R_3}{R_1}$$

Equating imaginary parts,

$$\omega L_x = R_2 R_3 \omega C_1$$

$$L_x = R_2 R_3 C_1$$

The quality factor of the coil is given by,

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_3 C_1}{\left(\frac{R_2 R_3}{R_1} \right)}$$

$$Q = \omega R_1 C_1$$

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The arms of an a.c. Maxwell's bridge are adjusted as :

Arm AB : non-reactive resistance of 700Ω

Arm CD : non-reactive resistance of 300Ω

Arm AD : non-reactive resistance of 1200Ω in parallel with capacitor of $0.5 \mu\text{F}$.

If the bridge is balanced under this condition, find the components of the branch BC.

Solution : The bridge is shown in the Fig.

From the bridge,

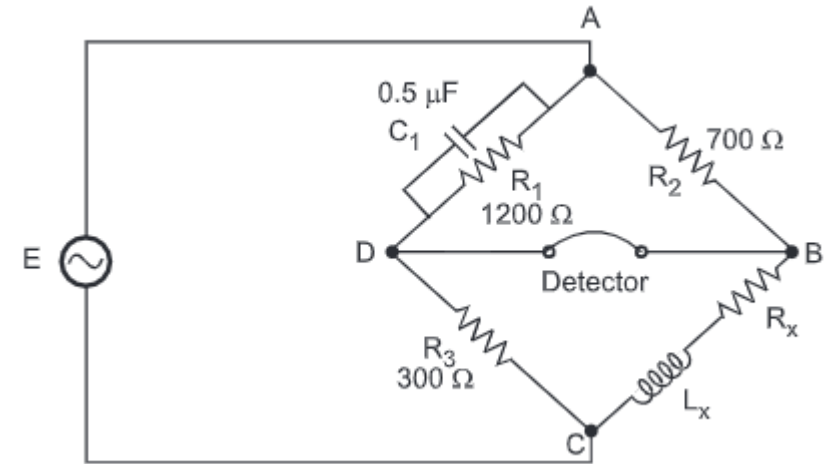
$$C_1 = 0.5 \mu\text{F}, \quad R_1 = 1200 \Omega$$

$$R_2 = 700 \Omega, \quad R_3 = 300 \Omega$$

From bridge balance equation,

$$R_x = \frac{R_2 R_3}{R_1} = \frac{700 \times 300}{1200} = 175 \Omega$$

And
$$L_x = R_2 R_3 C_1 = 700 \times 300 \times 0.5 \times 10^{-6} = 105 \text{ mH}$$



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Anderson Bridge

It is another important a.c. bridge used for the measurement of self inductance in terms of a standard capacitor.

Actually this bridge is modified Maxwell's bridge in

the value of self inductance is obtained by comparing it with a standard capacitor.

The Anderson bridge is as shown in the Fig.

One arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x .

This resistance R_1 includes resistance of the inductor.

C is the standard capacitor with r , R_2 , R_3 and R_4 are non-inductive known resistances.

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The bridge balance equations are,

$$i_1 = i_3,$$

$$i_2 = i_4 + i_C,$$

$$V_2 = i_2 R_2,$$

$$V_3 = i_3 R_3$$

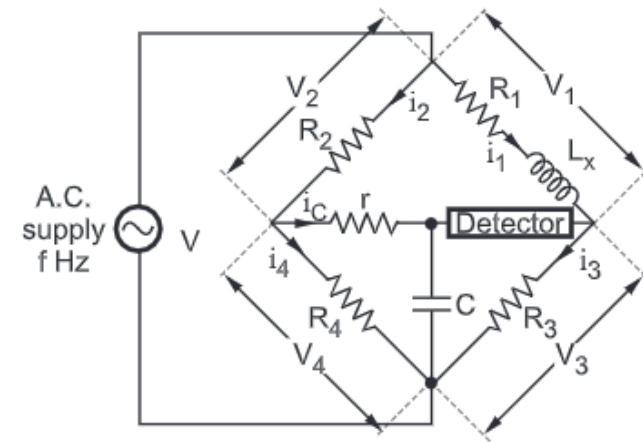
$$V_1 = V_2 + i_C r$$

$$V_4 = V_3 + i_C r,$$

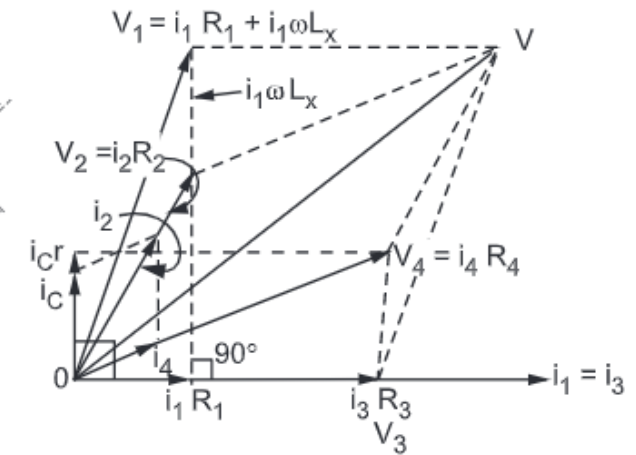
$$V_1 = i_1 R_1 + i_1 \omega L_1,$$

$$V_4 = i_4 R_4$$

$$V = \bar{V}_2 + \bar{V}_4 = \bar{V}_1 + \bar{V}_3$$



(a) Circuit diagram



(b) Phasor diagram

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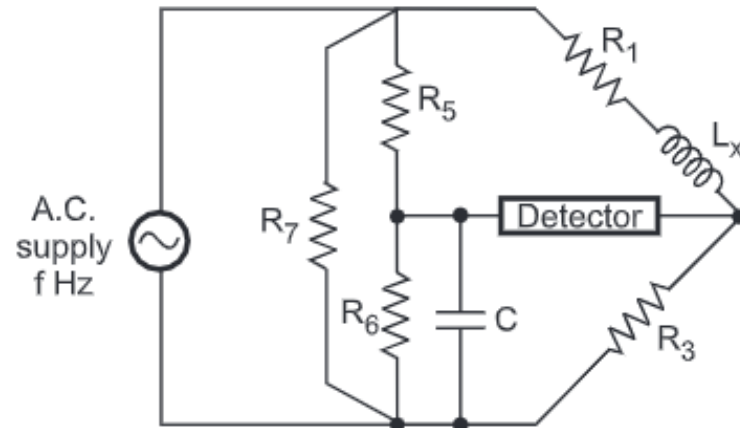
To find balance equations transforming a star formed by R_2 , R_4 and r into its equivalent delta as shown in the Fig. (a) and (b).

The elements in equivalent delta are given by,

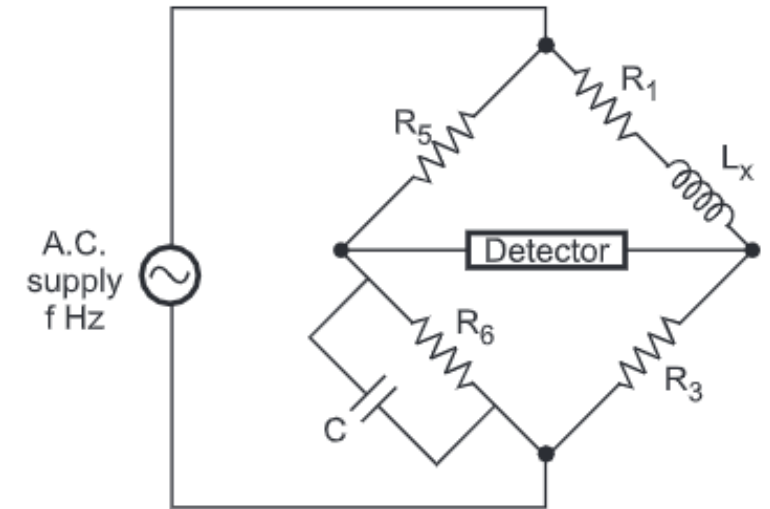
$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$



(a)



(b)

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Thus, balance equations are given by,

$$L_x = CR_3R_5$$

$$R_1 = R_3 \frac{R_5}{R_6}$$

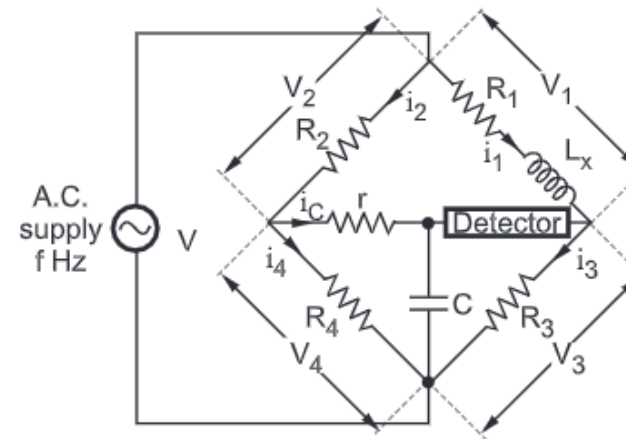
Substituting values of R_5 and R_6 , we can write,

$$L_x = \frac{CR_3}{R_4} [R_2r + R_4r + R_2R_4] \quad \text{and}$$

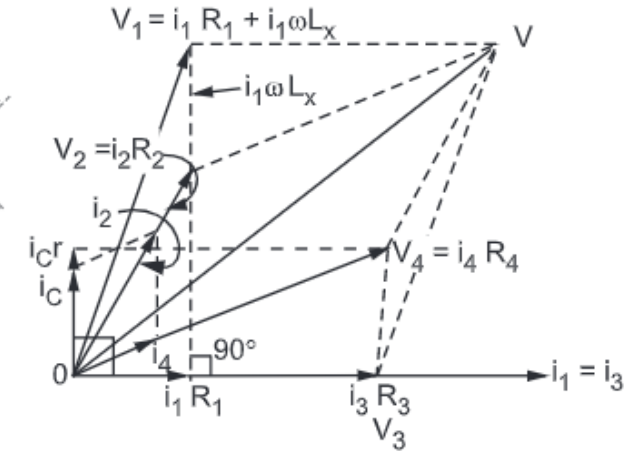
$$R_1 = \frac{R_2R_3}{R_4}$$

If the capacitor used is not perfect, the value of inductance remains unchanged, but the value of R_1 changes.

This method can also be used to measure the capacitance of the capacitor C if a calibrated self inductance is available.



(a) Circuit diagram



(b) Phasor diagram

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Advantages of Anderson Bridge

1. Can be used for accurate measurement of capacitance in terms of inductance.
2. Other bridges require variable capacitor but a fixed capacitor can be used for Anderson's bridge.
3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q .

Disadvantages of Anderson Bridge

1. It is more complicated than other bridges.
2. Uses more number of components.
3. Balance equations are also complicated to derive.
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

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An Anderson a.c. bridge is as follows :

Arm AB : Unknown inductance R_x and L_x ,

Arm BC : Non-reactive resistance $R_2 = 1000 \Omega$

Arm CD : Non-reactive resistance $R_4 = 1000 \Omega$

Arm DA : Non-reactive resistance $R_3 = 500 \Omega$

Arm DE : Resistance $r = 100 \Omega$

Arm EB : Detector and a.c. supply between AC

Arm EC : Capacitor $C = 3 \mu F$

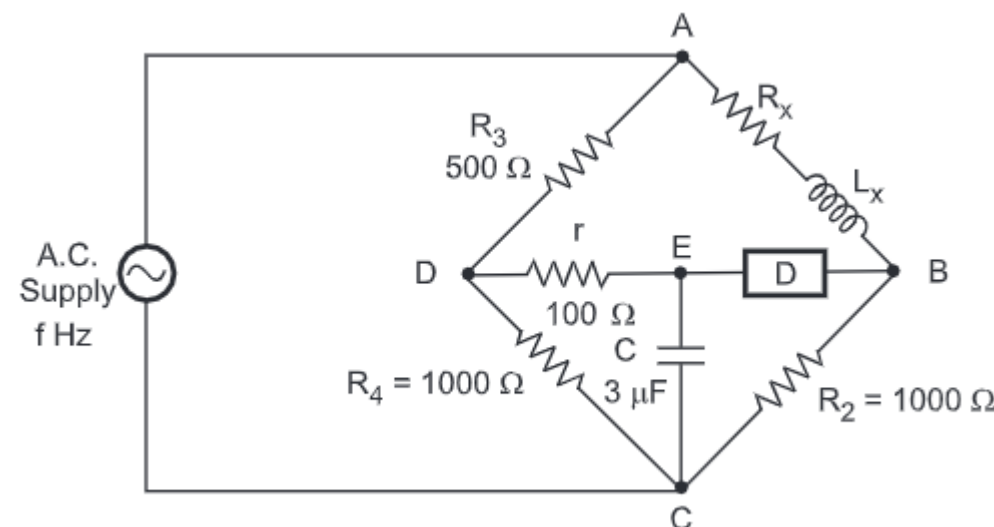
State the expressions for L_x and R_x and find the values of them for given values of elements.

Solution : The bridge is shown in the Fig.

$$R_x = \frac{R_2 R_3}{R_4} = \frac{1000 \times 500}{1000} = 500 \Omega$$

$$L_x = \frac{C R_2}{R_4} [R_3 r + R_4 r + R_3 R_4]$$

$$= \frac{3 \times 10^{-6} \times 1000}{1000} [500 \times 100 + 1000 \times 100 + 500 \times 1000] = 1.95 \text{ H}$$



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Hay's Bridge

The limitation of Maxwell's bridge is that it cannot be used for high Q values.

Hay's bridge is suitable for the coils having high Q values.

Hay's bridge consists of resistance R_1 in series with the standard capacitor C_1 in one of the ratio arms.

larger phase angles R_1 needed is very low, which is practicable.

Hence bridge can be used for the coils with high Q values.

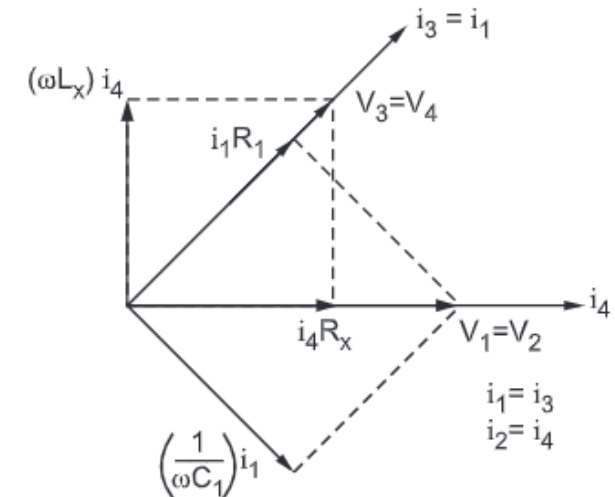
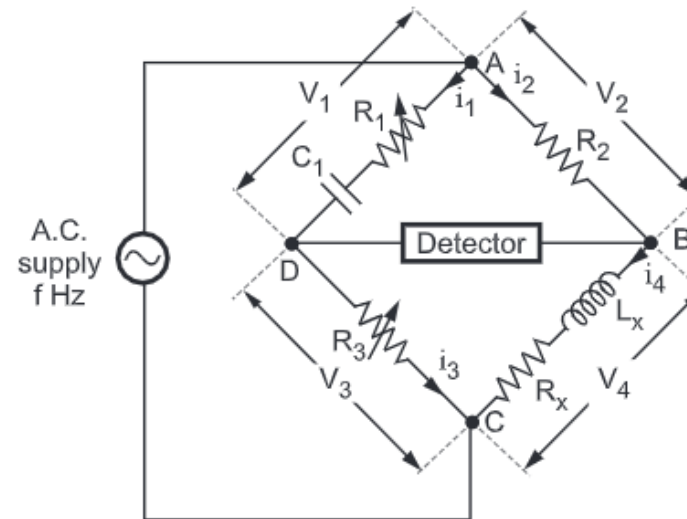
Under balanced condition,

The various constants of the bridge are :

$$Z_1 = R_1 - j X_{C_1} = R_1 - j \left(\frac{1}{\omega C_1} \right)$$

$$Z_2 = R_2 \quad \text{and} \quad Z_3 = R_3$$

$$Z_4 = Z_x = R_x + j (\omega L_x)$$



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At the balance condition,

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

$$\therefore \left[R_1 - j \left(\frac{1}{\omega C_1} \right) \right] [R_x + j (\omega L_x)] = R_2 R_3$$

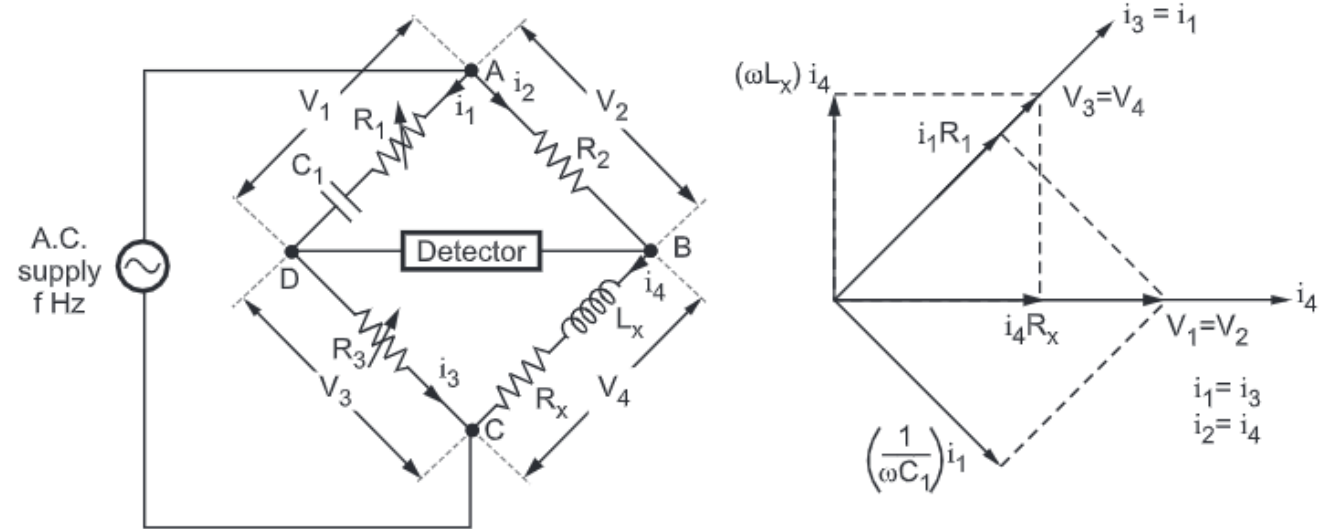
$$\therefore \left[R_x R_1 + \frac{L_x}{C_1} \right] + j \left[\omega R_1 L_x - \frac{R_x}{\omega C_1} \right] = R_2 R_3 \quad \dots (1)$$

Equating the real parts of both sides,

$$R_x R_1 + \frac{L_x}{C_1} = R_2 R_3 \quad \dots (2)$$

Equating the imaginary parts of both sides of equation (1),

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0 \quad \dots (3)$$



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To obtain R_x and L_x , solve equation (2) and equation (3) simultaneously.

From equation (3),

$$\omega R_1 L_x = \frac{R_x}{\omega C_1}$$

$$L_x = \frac{R_x}{\omega^2 R_1 C_1} \quad \dots (4)$$

Substituting in equation (2),

$$R_x R_1 + \frac{R_x}{\omega^2 R_1 C_1^2} = R_2 R_3$$

$$R_x \left[R_1 + \frac{1}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

$$R_x \left[\frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

$$R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2} \quad \dots (5)$$

Substituting equation (5) in equation (4) we get,

$$L_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{(1 + \omega^2 R_1^2 C_1^2) \omega^2 R_1 C_1}$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2} \quad \dots (6)$$

This is same as Maxwell's bridge equation.

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De Sauty Bridge

De Sauty bridge is to compare two capacitance.

The circuit diagram of De Sauty bridge is as shown in the Fig.

The De Sauty bridge consists a capacitor C_1 which is capacitance under test in branch AB.

The branch AD consists a known, standard capacitor.

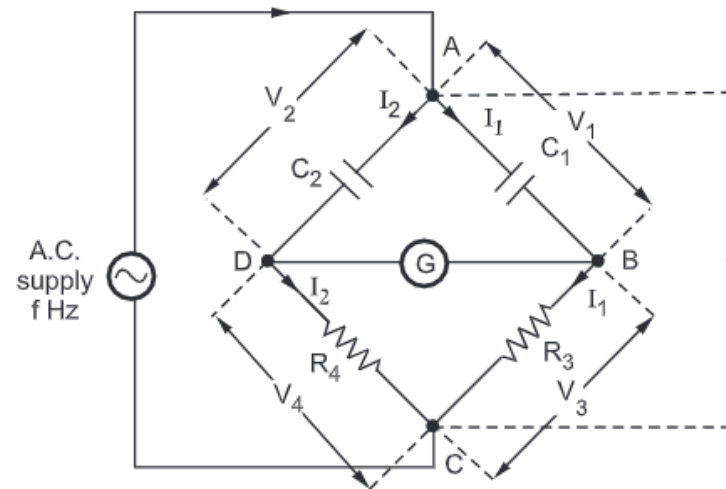
The remaining branches BC and CD consists non-inductive resistances R_3 and R_4 respectively.

At balance, we get condition as,

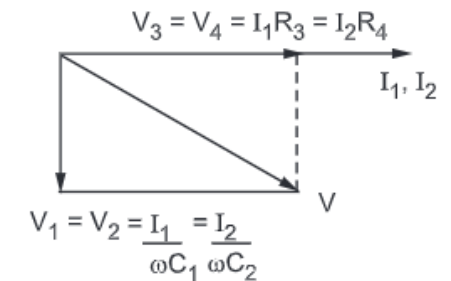
$$\frac{-jX_{C_1}}{R_3} = \frac{-jX_{C_2}}{R_4} \quad \dots (1)$$

$$\left(\frac{-j}{\omega C_1}\right) R_4 = \left(\frac{-j}{\omega C_2}\right) R_3$$

$$C_1 = \frac{C_2 \cdot R_4}{R_3} \quad \dots (2)$$



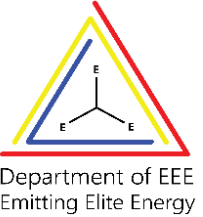
(a) Circuit diagram



(b) Phasor diagram



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The balance in the bridge can be achieved by varying either R_3 or R_4 . Under the balanced condition the vector diagram for the De Sauty bridge is as shown in the Fig.

The main advantage of De Sauty bridge is that it is very simple in construction and the measurement of the capacitor is done with simplicity. In spite of the simplicity of the measurement, it has one drawback that when the capacitors are with dielectric losses, then it is highly impossible to achieve balance condition. Thus practically De Sauty can be used effectively for the capacitors with very low dielectric losses such as air capacitors.

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Schering Bridge

It is one of the most widely used a.c. bridges for the measurement of unknown capacitors, dielectric loss and power factor.

It can be used for low voltages.

The C_x is perfect capacitor to be measured. R_x is series resistance. C_2 is standard air capacitor having very stable value. R_3 and R_4 are non-inductive resistances while C_4 is variable capacitor.

From the general balance equation,

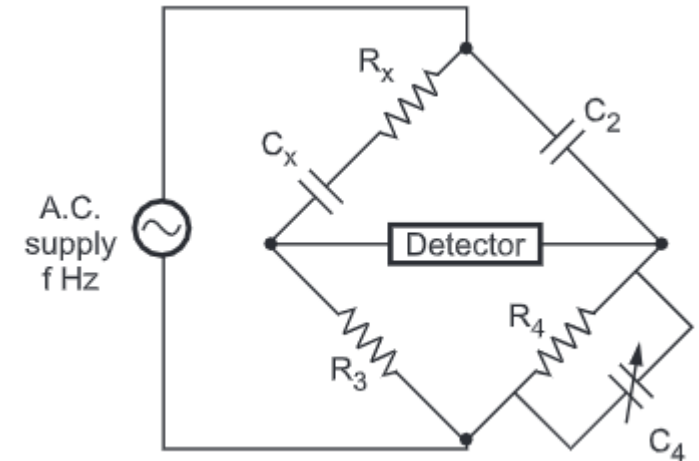
$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$Z_1 = R_x - j \frac{1}{\omega C_x}$$

$$Z_2 = - \frac{j}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{-j}{\omega C_4} = \frac{R_4 \left(-\frac{j}{\omega C_4} \right)}{\left(R_4 - j \frac{1}{\omega C_4} \right)}$$



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$$Z_4 = \frac{-j R_4}{\omega R_4 C_4 - j} = \frac{-j R_4 (\omega R_4 C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)} = \frac{R_4 - j \omega R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4} = \frac{\left(-\frac{j}{\omega C_2}\right)(R_3)}{\left(\frac{R_4 - j \omega R_4^2 C_4}{1 + \omega^2 R_4^2 C_4^2}\right)} = \frac{(1 + \omega^2 R_4^2 C_4^2) R_3 \left(-\frac{j}{\omega C_2}\right)}{(R_4 - j \omega R_4^2 C_4)}$$

$$\text{Rationalising, } Z_1 = R_3 (1 + \omega^2 R_4^2 C_4^2) \left\{ \frac{-\frac{j}{\omega C_2} (R_4 + j \omega R_4^2 C_4)}{R_4^2 + \omega^2 R_4^4 C_4^2} \right\}$$

$$\therefore R_x - j \frac{1}{\omega C_x} = \frac{R_3 (1 + \omega^2 R_4^2 C_4^2)}{R_4^2 (1 + \omega^2 R_4^2 C_4^2)} \left\{ \frac{R_4^2 C_4}{C_2} - \frac{j R_4}{\omega C_2} \right\}$$

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Equating real and imaginary parts,

$$R_x = \frac{R_3}{R_4^2} \times \frac{R_4^2 C_4}{C_2} = \frac{R_3 C_4}{C_2} \quad \dots (1)$$

$$-j \frac{1}{\omega C_x} = -j \frac{R_3}{R_4^2} \times \frac{R_4}{\omega C_2} = -j \left[\frac{1}{\frac{R_4}{R_3} \omega C_2} \right]$$

$$\omega C_x = \frac{R_4}{R_3} \omega C_2$$

$$C_x = \frac{R_4}{R_3} C_2 \quad \dots (2)$$

The equations (1) and (2) gives the required values of C_x and R_x .

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Power Factor and Loss Angle

- i) **Power factor (p.f.)** : The power factor of the series RC combination is defined as the cosine of the phase angle of the circuit. Thus,

$$\text{p.f.} = \cos \phi_x = \frac{R_x}{Z_x}$$

For phase angles very close to 90° , the reactance is almost equal to the impedance,

$$\therefore \text{p.f.} = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)}$$

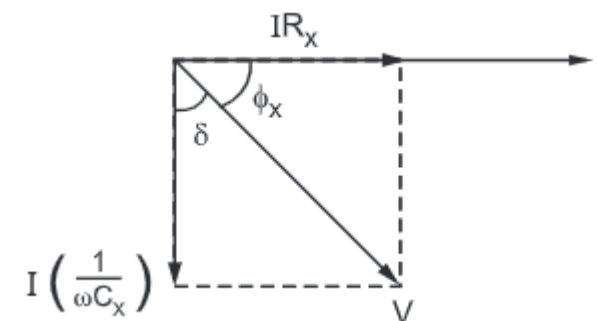
$$\text{p.f.} = \omega R_x C_x$$

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ii) Loss angle (δ) : For a series combination of R_x and C_x , the angle between the voltage across the series combination and voltage across the capacitor C_x is called **loss angle δ** .

$$\text{Now } \tan \delta = \frac{I R_x}{I \left(\frac{1}{\omega C_x} \right)} = \omega R_x C_x$$

$$\therefore \tan \delta = \omega \left(\frac{R_3 C_4}{C_2} \right) \left(\frac{R_4}{R_3} C_2 \right) = \omega R_4 C_4$$



Thus loss angle can be measured, knowing the values of ω , R_4 and C_4 .

iii) Dissipation factor (D) : For R_x - C_x series circuit, it is cotangent of the phase angle ϕ_x .

$$D = \cot \phi_x = \frac{1}{\tan \phi_x} = \frac{1}{\left[\frac{I \left(\frac{1}{\omega C_x} \right)}{I R_x} \right]} = \omega R_x C_x = \omega R_4 C_4$$

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The Schering bridge has the following constants :

Arm AB - capacitor of $1 \mu\text{F}$ in parallel with $1.2 \text{ k}\Omega$ resistance

Arm AD - resistance of $4.7 \text{ k}\Omega$

Arm BC - capacitor of $1 \mu\text{F}$

Arm CD - unknown capacitor C_x and R_x .

The frequency of supply is 0.5 kHz . Calculate the unknown capacitance and its dissipation factor.

Solution : From the given information,

$$R_1 = 1.2 \text{ k}\Omega \quad C_1 = 1 \mu\text{F}$$

$$R_2 = 4.7 \text{ k}\Omega \quad C_3 = 1 \mu\text{F}$$

From the balance equations,

$$R_x = \frac{R_2 C_1}{C_3} = \frac{4.7 \times 10^3 \times 1 \times 10^{-6}}{1 \times 10^{-6}} = 4.7 \text{ k}\Omega$$

$$C_x = \frac{R_1 C_3}{R_2} = \frac{1.2 \times 10^3 \times 1 \times 10^{-6}}{4.7 \times 10^3} = 0.255 \mu\text{F}$$

The dissipation factor,

$$\begin{aligned} D &= \omega C_x R_x = 2\pi f C_x R_x \\ &= 2\pi \times 0.5 \times 10^3 \times 0.255 \times 10^{-6} \times 4.7 \times 10^3 \\ &= 3.765 \end{aligned}$$

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Given the Maxwell bridge as shown in the Fig. find the equivalent series resistance and inductance of R_x and L_x at balance.

$$Z_1 = R_1 \parallel X_{C1}$$

$$Y_1 = \frac{1}{R_1} + j \omega C_1$$

$$= \frac{1}{600} + j (2\pi \times 1000 \times 1 \times 10^{-6})$$

$$= 1.66 \times 10^{-3} + j 6.283 \times 10^{-3}$$

$$Z_2 = 100 \Omega$$

$$Z_3 = 1000 \Omega$$

$$Z_4 = R_x + j X_L = Z_x$$

From the basic balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

$$= 100 \times 1000 \times [1.66 \times 10^{-3} + j 6.283 \times 10^{-3}]$$

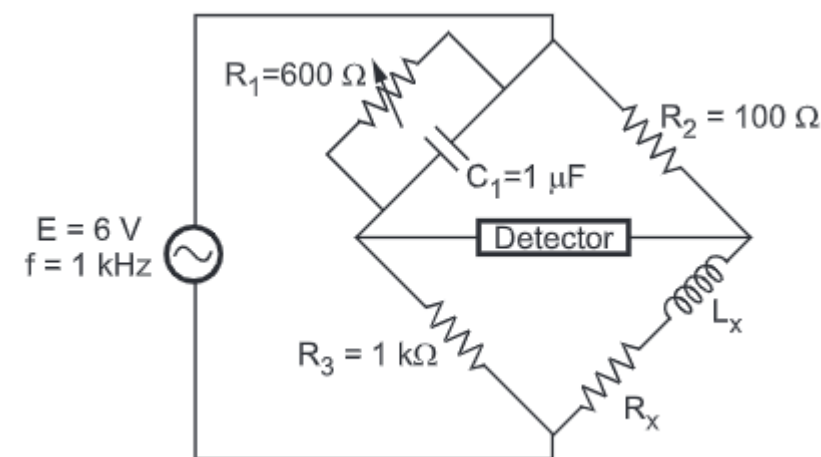
$$= 166 + j 628.3 \Omega = R_x + j X_L \Omega$$

$$R_x = 166 \Omega$$

$$X_L = 628.3 = 2\pi f L_x$$

$$L_x = \frac{628.3}{2\pi \times 1000}$$

$$= 0.099 \text{ H}$$



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An a.c. bridge circuit for measurement of effective inductance and capacitance of an iron cored coil is as follows : Arm AB : the unknown impedance, Arm BC : a pure resistance of $10\ \Omega$, Arm CB : a loss free capacitance of $1\ \mu\text{F}$ and Arm AD : a capacitance of $0.135\ \mu\text{F}$ in series with $842\ \Omega$ resistance. Obtain the balance equations of the bridge and determine the unknown parameters in the arm AB.

The general balance equation is,

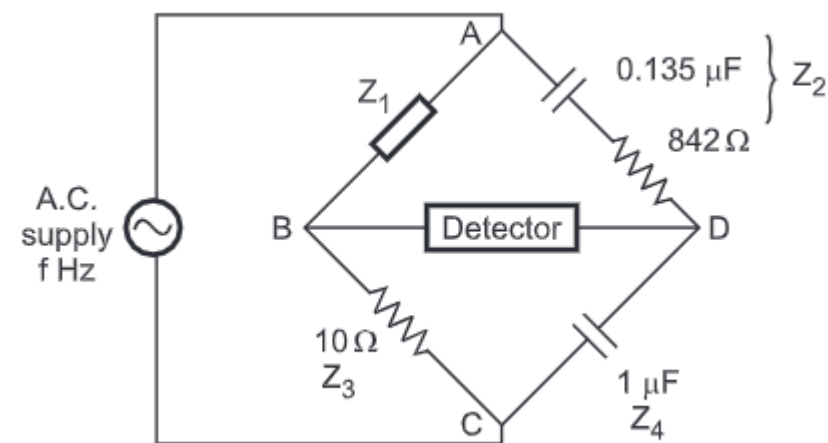
$$\therefore \quad \boxed{Z_1 Z_4 = Z_2 Z_3}$$

$$\boxed{Z_1 = \frac{Z_2 Z_3}{Z_4}}$$

Let the frequency is ω rad/sec.

$$Z_2 = \left[842 - j \frac{1}{\omega 0.135 \times 10^{-6}} \right] = 842 - j \frac{7.4074 \times 10^6}{\omega} \ \Omega$$

$$Z_3 = 10 + j 0 = 10 \angle 0^\circ \ \Omega$$



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$$Z_4 = 0 - j \frac{1}{\omega \times 1 \times 10^{-6}} = -j \frac{10^6}{\omega} = \frac{10^6}{\omega} \angle -90^\circ \Omega$$

$$Z_1 = \frac{\left[842 - j \frac{7.4074 \times 10^6}{\omega} \right] [10]}{\frac{10^6}{\omega} \angle -90^\circ} = \left[8420 - j \frac{7.4076 \times 10^7}{\omega} \right] \frac{1}{10^6} \angle +90^\circ$$

$$= \left[8420 - j \frac{7.4076 \times 10^7}{\omega} \right] \left[+j \frac{\omega}{10^6} \right] = j \omega \frac{8420}{10^6} + \frac{7.4076 \times 10^7}{10^6}$$

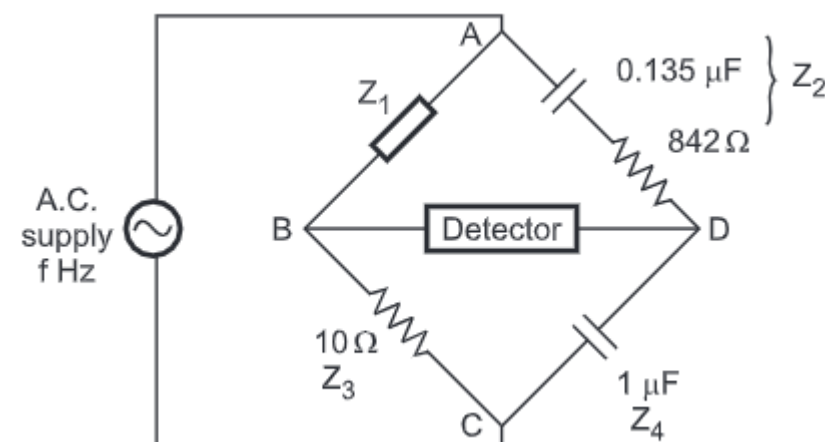
$$= 74.076 + j \omega 8420 \times 10^{-6}$$

Comparing Z_1 with $R_1 + j \omega L_1$

$$\therefore R_1 = 74.076 \Omega$$

$$L_1 = 8420 \mu\text{H} = 8.42 \text{ mH.}$$

These are the unknown parameters of arm AB.



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An a.c. bridge circuit working at 1 kHz have its arms as follows.

Arm AB : $0.2 \mu\text{F}$ capacitance Arm BC : 500Ω resistance

Arm CD : unknown impedance

Arm DA : 300Ω resistance in parallel with $0.1 \mu\text{F}$ capacitor.

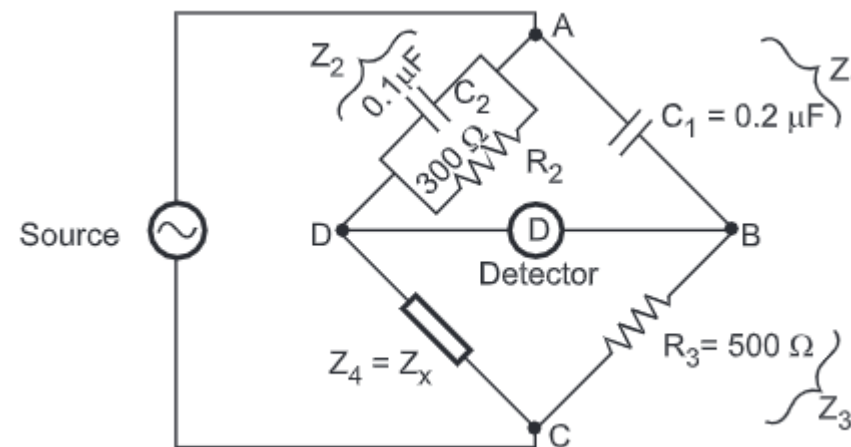
Find R and L or C constants of the arm CD considering it as a series circuit.

Consider the basic a.c. bridge as shown in the Fig.

The branch AB consists $C_1 = 0.2 \mu\text{F}$. Hence impedance of branch AB i.e. Z_1 is given by,

$$Z_1 = -jX_{C_1} = -j\frac{1}{\omega C_1} = \frac{-j}{2\pi f C_1} = \frac{-j}{2 \times \pi \times 1 \times 10^3 \times 0.2 \times 10^{-6}}$$

$$Z_1 = -j795.77 \Omega = 795.77 \angle -90^\circ \Omega$$



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Similarly, branch AD consists parallel combination of $R_2 = 300 \Omega$ and $C_2 = 0.1 \mu F$. Hence, impedance of branch AD i.e. Z_2 is given by,

$$Z_2 = R_2 || -jX_{C_2} = R_2 || \left(\frac{-j}{\omega C_2} \right) = \frac{R_2 \left(\frac{-j}{\omega C_2} \right)}{R_2 - \frac{j}{\omega C_2}}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2} = \frac{R_2}{1 + j(2\pi \times f \times C_2 \times R_2)} = \frac{300}{1 + j(2\pi \times 1 \times 10^3 \times 0.1 \times 10^{-6} \times 300)} = \frac{300}{1 + j 0.1885}$$

$$= \frac{300}{1.0176 \angle + 10.695^\circ}$$

$$Z_2 = 294.81 \angle -10.67^\circ \Omega$$

For balance, the condition is given by,

$$Z_4 = \frac{Z_2 \cdot Z_3}{Z_1}$$

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Substituting values of Z_1 , Z_2 and Z_3 , we get,

$$Z_4 = \frac{(294.81 \angle -10.67^\circ) (500)}{795.77 \angle -90^\circ}$$

$$Z_4 = 185.2356 \angle 79.33^\circ \Omega$$

$$Z_4 = R_4 + j X_{L_4} = (34.2967 + j 182.03) \Omega$$

Thus inductive reactance can be written as,

$$X_{L_4} = \omega L_4 = 2\pi f L_4 = 182.03$$

$$\therefore 2 \times \pi \times 1 \times 10^3 \times L_4 = 182.03$$

$$L_4 = 28.97 \text{ mH}$$

$$R_4 = 34.2967 \Omega$$

Thus branch CD is a series R-L circuit consisting $R_4 = 34.2967 \Omega$ and $L_4 = 28.97 \text{ mH}$.

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A 4 terminal resistance of approximately $50 \mu\Omega$ was measured with the help of Kelvin double bridge under the following conditions

Value of standard resistance $100.03 \mu\Omega$;

Resistance of inner ratio arms 100.31Ω and 200Ω

Resistance of outer ratio arms 100.24Ω and 200Ω ;

Value of low resistance link $700 \mu\Omega$

Calculate the magnitude of error in the measurements.

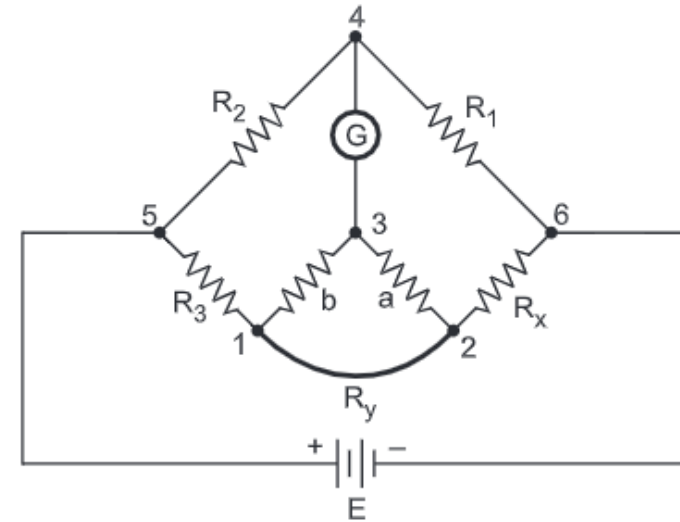
A Kelvin double bridge can be drawn as shown in the Fig.

By using the expression of unknown resistance we can write,

$$R_x = \frac{R_1}{R_2} R_3 + \frac{bR_y}{a+b+R_y} \left[\frac{R_1}{R_2} - \frac{a}{b} \right]$$

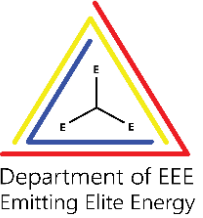
$$R_x = \frac{100.24}{200} (100.03 \times 10^{-6}) + \frac{(200)(700 \times 10^{-6})}{100.31 + 200 + 700 \times 10^{-6}} \left[\frac{100.24}{200} - \frac{100.31}{200} \right]$$

$$R_x = 49.97 \times 10^{-6} \Omega = 49.97 \mu\Omega$$





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Hence the magnitude of error in the measurement is given by,

$$\begin{aligned} |\text{error}| &= |\text{Actual value of resistance} - \text{Measured value of resistance}| \\ &= |50 \times 10^{-6} - 49.97 \times 10^{-6}| = 0.03 \times 10^{-6} \end{aligned}$$

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The four impedances of an a.c. bridge are

$$Z_{AB} = 400 \angle 50^\circ \Omega, \quad Z_{AD} = 200 \angle 40^\circ \Omega,$$

$$Z_{BC} = 800 \angle -50^\circ \Omega, \quad Z_{CD} = 400 \angle 20^\circ \Omega$$

Find out whether the bridge is balanced under these conditions or not.

Solution : For an a.c. bridge, the balance conditions are given by,

$$Z_1 Z_4 = Z_2 Z_3$$

... condition of balance for magnitudes

$$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$

... condition of balance for phases.

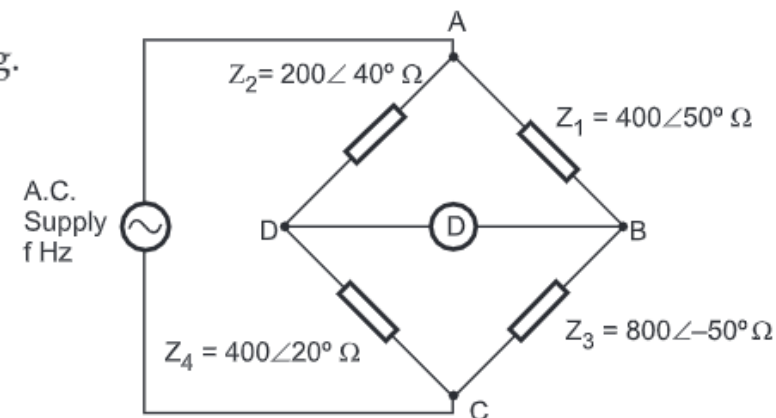
Consider the basic a.c. bridge with four impedances as shown in the Fig.

Applying the condition of balance for the magnitudes, we get,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$(400)(400) = (200)(800)$$

That means condition of balance for magnitudes is satisfied.



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Applying the condition of balance for phases, we get

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

$$\text{L.H.S.} = \angle\theta_1 + \angle\theta_4 = [50^\circ + 20^\circ] = 70^\circ$$

$$\text{R.H.S.} = \angle\theta_2 + \angle\theta_3 = [40^\circ - 50^\circ] = -10^\circ$$

As the values on L.H.S. and R.H.S. of equation (b) are not equal, **the condition of balance for phases is not satisfied.**

Thus for above given conditions, the bridge is in unbalanced condition because eventhough condition of balance for magnitudes is satisfied; condition of balance for phases is not satisfied.

A four arm a.c. bridge a-b-c-d has following impedances.

Arm ab : $Z_1 = 200 \angle 60^\circ \Omega$, Arm ad : $Z_2 = 400 \angle -60^\circ \Omega$

Arm bc : $Z_3 = 300 \angle 0^\circ \Omega$, Arm cd : $Z_4 = 600 \angle 30^\circ \Omega$

Determine whether it is possible to balance the bridge under above conditions.

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The arms of five node bridge are as follows :

Arm ab : an unknown impedance (R_1, L_1) in series with a non-variable resistor r_1 .

Arm bc : a non-inductive resistor $R_3 = 100 \Omega$.

Arm cd : a non-inductive resistor $R_4 = 200 \Omega$.

Arm da : a non-inductive resistor $R_2 = 250 \Omega$.

Arm de : a variable non-inductive resistor r .

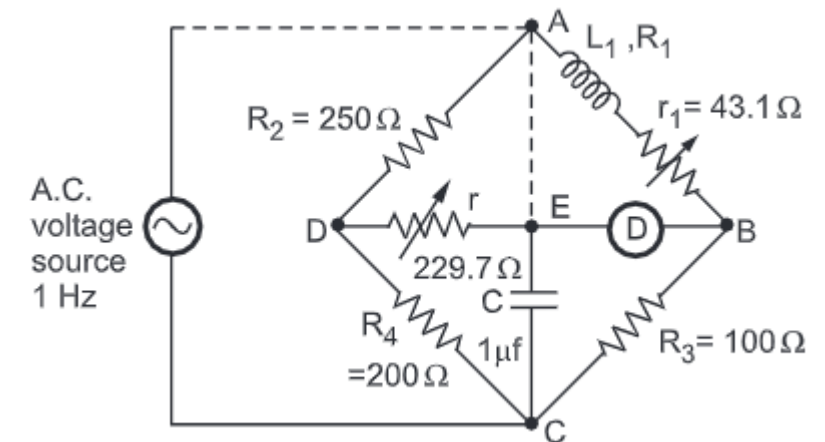
Arm ec : a lossless capacitor $C = 1 \mu F$.

An a.c. supply is connected between a and c. Detector is between b and e. Calculate the resistance R_1 and inductance L_1 when under balance condition $r_1 = 43.1 \Omega$ and $r = 229.7 \Omega$.

Solution : The a.c. bridge is as shown in the Fig.

From circuit arrangement, it is Anderson's bridge. The value of unknown resistor is given by,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1 = \frac{(250)(100)}{200} - 43.1 = 81.9 \Omega$$



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Unknown inductance is given by,

$$\begin{aligned} L_1 &= \frac{CR_3}{R_4} [(R_2 + R_4) r + R_2 R_4] \\ &= \frac{1 \times 10^{-6} \times 100}{200} [(250 + 200) (229.7) + (250) (200)] \\ &= 0.0766 \text{ H} = 76.6825 \text{ mH} \end{aligned}$$

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The four arms of the bridge are as follows :

Arm ab : An imperfect capacitor C_1 with an equivalent series resistance of r_1

Arm bc : A non-inductive resistance R_3

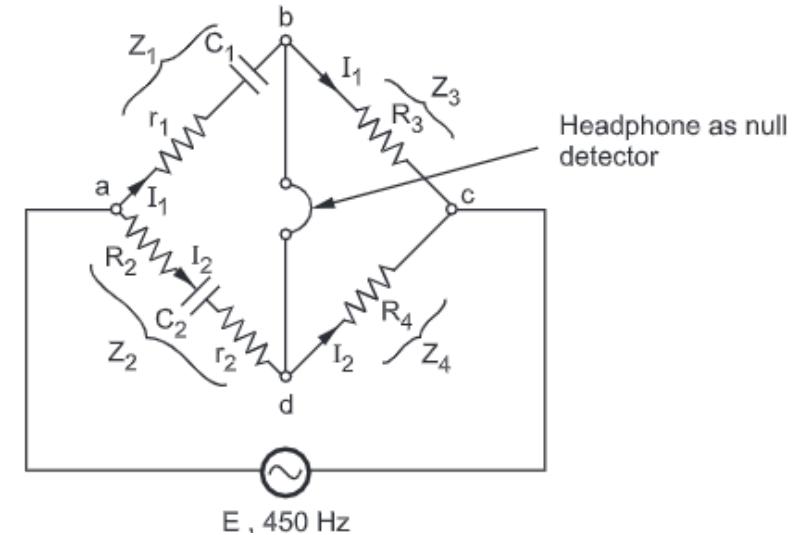
Arm cd : A non-inductive resistance R_4

Arm da : An imperfect capacitor C_2 with an equivalent resistance of r_2 in series with resistance R_2 .

A supply at 450 Hz is connected between terminals a and c and the detector is connected between b and d. At the balance condition :

$R_2 = 4.8 \, \Omega$, $R_3 = 200 \, \Omega$, $R_4 = 2850 \, \Omega$, and $C_2 = 0.5 \, \mu\text{F}$, $r_2 = 0.4 \, \Omega$

Calculate values of C_1 and r_1 and also of the dissipating factor for the capacitor.



Solution : The bridge is as shown in the Fig.

$$Z_1 = r_1 - j \frac{1}{\omega C_1} \, \Omega$$

$$Z_2 = (R_2 + r_2) - j \frac{1}{\omega C_2} = (4.8 + 0.4) - j \frac{1}{2\pi \times 450 \times 0.5 \times 10^{-6}} = 5.2 - j 707.3553 \, \Omega = 707.3744 \angle -89.5788^\circ \, \Omega$$

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$$Z_3 = 200 + j 0 \, \Omega = 200 \angle 0^\circ \, \Omega$$

$$Z_4 = 2850 + j 0 \, \Omega = 2850 \angle 0^\circ \, \Omega$$

At a bridge balance, no current flows through the detector.

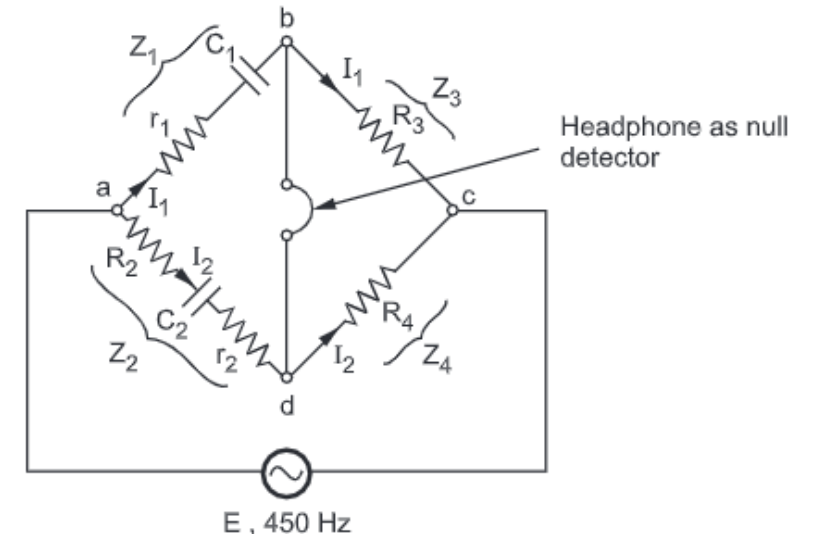
$$\therefore I_1 = \frac{E}{Z_1 + Z_3}$$

$$I_2 = \frac{E}{Z_2 + Z_4}$$

Now $I_1 Z_1 = I_2 Z_2$ for null deflection of detector

$$\therefore \frac{E Z_1}{Z_1 + Z_3} = \frac{E Z_2}{Z_2 + Z_4}$$

$$\therefore Z_1 Z_4 = Z_2 Z_3$$



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$$\therefore 2850 \left[r_1 - j \frac{1}{\omega C_1} \right] = 200 \angle 0^\circ \times 707.3744 \angle -89.5788^\circ$$

$$\therefore r_1 - j \frac{1}{\omega C_1} = 49.6403 \angle -89.5788^\circ = 0.3649 - j 49.6389 \, \Omega$$

Comparing both sides,

$$r_1 = 0.3649 \, \Omega \quad \text{and} \quad \frac{1}{\omega C_1} = 49.6389$$

$$C_1 = \frac{1}{2\pi \times 450 \times 49.6389} = 7.125 \, \mu\text{F}$$

$$\begin{aligned} \text{Dissipating factor} &= \omega r_1 C_1 = 2\pi \times 450 \times 0.3649 \times 7.125 \times 10^{-6} \\ &= 0.007351 \end{aligned}$$

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The four arms of the Maxwell's capacitance bridge at balance are :

Arm ab : unknown inductance L_1 having an inherent resistance R_1

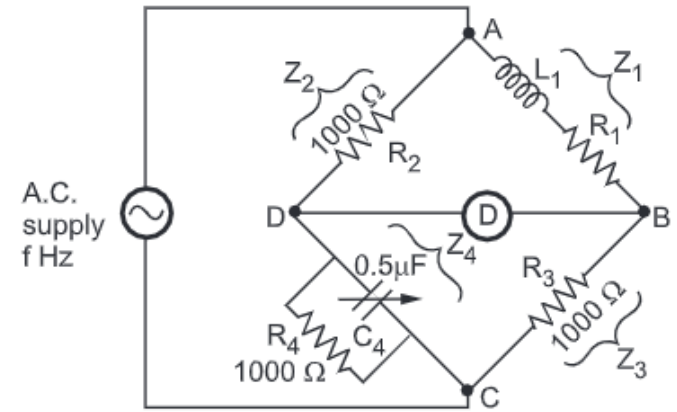
Arm bc : A non-inductive resistance of 1000Ω ,

Arm cd : A capacitor of $0.5 \mu F$ in parallel with a resistance of 1000Ω .

Arm da : A resistance of 1000Ω

Determine the values of R_1 and L_1 . Draw the phasor diagram of the bridge.

Solution : From the given information, the Maxwell's capacitance bridge is as shown



The equation for balance is,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$[R_1 + j\omega L_1] \left[\frac{R_4}{1 + j\omega C_4 R_4} \right] = (R_2) (R_3)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 R_4 C_4$$

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Equating real terms, we get,

$$R_1 R_4 = R_2 R_3$$
$$\therefore R_1 = \frac{R_2 R_3}{R_4} = \frac{(1000)(1000)}{1000} = 1000 \, \Omega$$

Equating imaginary terms, we get

$$\omega L_1 R_4 = \omega R_2 R_3 R_4 C_4$$

$$L_1 = R_2 R_3 C_4 = (1000)(1000) (0.5 \times 10^{-6}) = 0.5 \, \text{H}$$

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The four arms of Hay's bridge are arranged as follows :

AB : coil of unknown impedance.

BC : non-reactive resistance of 100Ω .

CD : non-reactive resistance of 833Ω in series with $0.38 \mu\text{F}$ capacitor.

DA : non-reactive resistor of 16800Ω .

If the supply frequency is 50 Hz , determine the inductance and resistance at the balance condition.

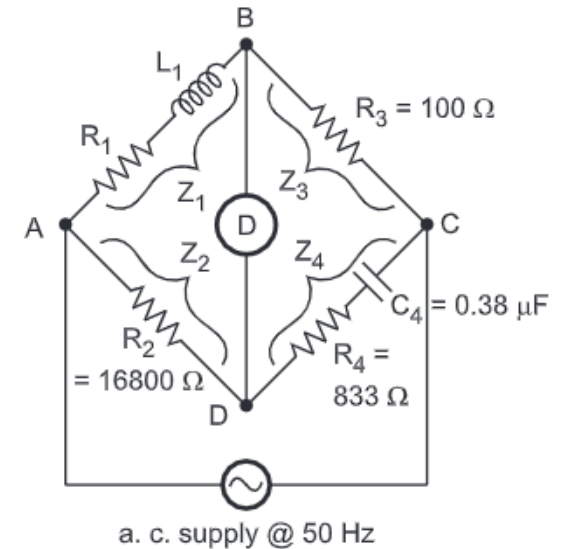
From given data, Hay's bridge can be drawn as shown in the Fig.

At $f = 50 \text{ Hz}$, reactance offered by C_4 is,

$$-jX_{C_4} = \frac{-j}{\omega C_4} = \frac{-j}{2\pi f C_4}$$

$$= \frac{-j}{2 \times \pi \times 50 \times 0.38 \times 10^{-6}}$$

$\therefore -jX_{C_4} = -j 8376.5759 \Omega$



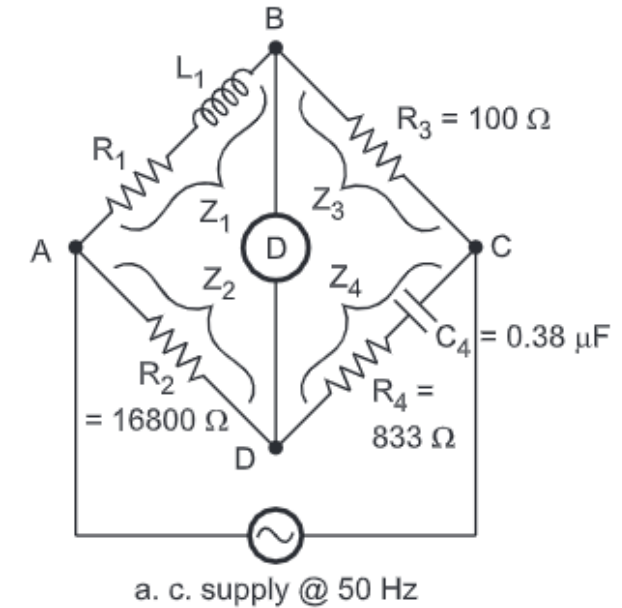
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$$\begin{aligned}\therefore Z_4 &= R_4 - jX_{C_4} \\ &= 833 - j 8376.5759 \\ \therefore Z_4 &= 8417.8924 \angle -84.32^\circ\end{aligned}$$

Now in general for a.c. bridge, the condition of balance is given by,

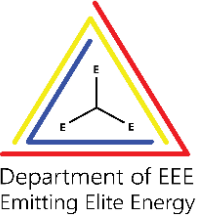
$$\begin{aligned}Z_1 Z_4 &= Z_2 Z_3 \\ Z_1 &= \frac{Z_2 Z_3}{Z_4} = \frac{(16800)(100)}{8417.8929 \angle -84.32^\circ} = 199.5749 \angle +84.32^\circ\end{aligned}$$

$$Z_1 = R_1 + jX_{L_1} = R_1 + j\omega L_1 = R_1 + j2\pi f L_1 = 19.7524 + j 198.595 \Omega$$





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Comparing imaginary terms on both the sides,

$$2\pi f L_1 = 198.595$$

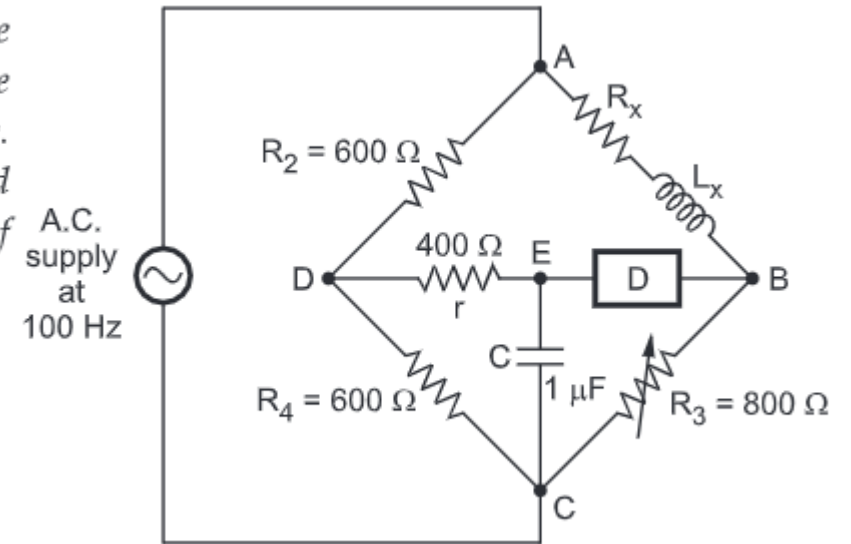
$$L_1 = \frac{198.595}{2\pi f} = \frac{198.595}{2 \times \pi \times 50} = \mathbf{0.6321 \text{ H}}$$

Comparing real terms on both the sides,

$$R_1 = \mathbf{19.7524 \Omega}$$

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In an Anderson's bridge for the measurement of inductance, the arm AB consists of an unknown impedance with inductance L and R , a unknown variable resistance in arm BC, fixed resistance of $600\ \Omega$ each in arms CD and DA, a unknown variable resistance in arm DE and a capacitor with fixed capacitance of $1\ \mu\text{F}$ in the arm CE. The a.c. supply of $100\ \text{Hz}$ is connected across A and C and the detector is connected between B and E. If the balance is obtained with a resistance of $400\ \Omega$ in the arm DE and a resistance of $800\ \Omega$ in the arm BC, calculate the values of R and L .



By the standard formula, the unknown resistance R_x in branch AB is given by,

$$R_x = R_1 = \frac{R_2 R_3}{R_4} = \frac{(600)(800)}{(600)} = 800\ \Omega$$

Similarly the unknown inductance in branch AB is given by,

$$L_x = \frac{C R_3}{R_4} [R_2 r + R_4 r + R_2 R_4] = \frac{(1 \times 10^{-6})(800)}{600} [(600)(400) + (600 \times 400) + (600)(600)] = 1.12\ \text{H}$$